(Proc 9th DOD/NASA/FAA Conf. On Fibrous Composites in Structural Design III, 1447 (1991)

ON THE THERMALLY-INDUCED RESIDUAL STRESSES IN THICK FIBER-THERMOPLASTIC MATRIX (PEEK) CROSS-PLY LAMINATED PLATES

Shoufeng Hu and John A. Nairn*
Materials Science and Engineering, University of Utah, Salt Lake City, Utah

SUMMARY

An analytical method for calculating thermally-induced residual stresses in laminated plates is applied to cross-ply PEEK laminates. We considered three cooling procedures — slow cooling (uniform temperature distribution), convective and radiative cooling, and rapid cooling by quenching (constant surface temperature). Some of the calculated stresses are of sufficient magnitude to effect failure properties such as matrix microcracking.

INTRODUCTION

Thermally-induced residual stresses in laminated composites are introduced by fabrication and by environmental exposure. They are an unavoidable consequence of (1) the nonuniform distribution of cooling temperature due to the phenomenon of heat transfer and (2) the difference in thermal expansion coefficients of lamina in the fiber direction and the transverse direction. For thin laminated plates the residual stresses caused by (1) may be ignored. But, for thick laminated plates the residual stresses caused by (1) can be as large as those caused by (2). Tensile residual stresses in off-axis plies (*e.g.* 90° plies) are particularly important because they may be large enough to promote damage by matrix microcracking. The prediction and measurement of residual stresses are therefore important topics that are relevant to production, design, and performance of composite components.

Residual (or thermal) stresses and heat transfer are classical problems for conventional materials. A number of investigations specific to composite materials are available (*e.g.* Refs. [1-6]). The theoretical and experimental investigations for residual stresses in Refs. [1-4] and [6] illustrate the residual stresses due to unequal thermal expansion coefficients in the fiber and the transverse directions. Using the finite difference method (for temperature) and finite element method (for thermal stresses), Chen *et. al.* [5] studied the failure of laminates under thermal and mechanical loading with the consideration of heat transfer. But only few investigators have considered the residual stresses caused by the nonuniform distribution of temperature, which is especially significant for thick laminates. The goal of present study is to gain insight into the mechanisms of thermally-induced residual stresses in cross-ply laminates, which are caused by both disparate thermal expansion coefficients and by nonuniform distribution of temperature during cooling.

Thermoplastic matrix (PEEK) composites have received much attention due to their high stiffness and high fracture toughness. The stress-free temperature in PEEK composites was measured to be about 310°C [1,6]. The processing temperature, melting temperature, and crystallization temperature are all above 310°C. We therefore treat PEEK as being fully crystallized at the stress-free temperature and calculate the residual stresses that develop on cooling from the stress-free temperature to room temperature. The problem can be separated into two discrete parts. The first part is the analysis of temperature distribution and the second part is the development of residual stresses for given temperature distribution. The problem is separable because heat transfer is not affected by the presence of residual stresses. We use a coordinate system centered inside the cross-ply laminates having the x-axis aligned with the fiber direction of top ply group and the z -axis perpendicular to plane of the plate.

* Supported in part by a NASA contract NAS1-18833

Our goal is to study the effect of heat transfer on the distribution of the thermally-induced residual stresses. To achieve this goal we consider three cooling procedures: 1) slow cooling in which temperature is uniform over entire thickness and residual stresses are uniform over each laminate group (*i.e*. heat transfer is ignored); 2) cooling under room temperature air by convection and radiation; 3) cooling after the surface is "quenched" to room temperature, (*i.e*. the surface temperature is equal to room temperature). The first and third cases provide two extreme conditions: infinitely large thermal conductivity (slow cooling) and infinitely large convection (or radiation) coefficient (constant surface temperature).

PART ONE — HEAT TRANSFER

To solve the problem analytically, we use the following assumptions: 1) Heat convection and radiation are assumed to take place only in the thickness direction. Any heat transfer around the edge is neglected. This assumption reduces the analysis to a one-dimensional problem. 2) Although the thermal conductivity, k, mass density, ρ, and the specific heat, C_p, all are functions of temperature, the ratio $k/\rho C_p$ is assumed to be temperature independent. 3) Linearization of thermal boundary conditions is assumed to be acceptable.

The governing differential equation for heat conduction without any inside heat source (Fourier equation) is [7]

$$\frac{\partial}{\partial z}\left[k(T)\frac{\partial T}{\partial z}\right] = \rho(T)C_p(T)\frac{\partial T}{\partial t}$$

or when the ratio $k/\rho C_p$ is independent of temperature

$$\alpha\frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t} \tag{1}$$

where $\alpha = k/\rho C_p$. The analysis of slowly cooled laminates does not involve any heat conduction analysis. The analyses of laminates cooled by convection and radiation or by quenching both use Eq. (1), but require different boundary conditions.

The heat convection and radiation boundary condition has the form

$$-k\frac{\partial T(d,t)}{\partial z} = I_r + I_c \tag{2}$$

where d is the half thickness of the plate, I_r and I_c are the surface energy losses due to radiation and convection, respectively, which are normally assumed to be

$$I_r = \sigma\varepsilon[T_{su}^4 - T^4(d,t)] \qquad \text{(Stefan-Boltzmann law)} \tag{3}$$

$$I_c = h_c[T_r - T(d,t)] \qquad \text{(Newton cooling law)} \tag{4}$$

where h_c is the convection coefficient; T_r is the air recovering temperature; σ is the Stefan-Boltzmann constant; ε is the surface emissivity of a "gray body" (instead of "black body"); and T_{su} is the temperature of an object surrounding the composite plate and receives the radiated heat. Because h_c and T_r are very complex functions of surface temperature, T(d,t) [5], both the convection and radiation parts of this boundary condition are nonlinear. To solve the problem analytically, we have to "linearize" the boundary condition. We linearize the convection part by assuming h_c to be temperature independent and letting T_r equal room temperature — T_0. We linearize the radiation part by letting $T_{su} = T_0$ and using the following simplification

$$I_r = \sigma\varepsilon[T_0^4 - T^4(d,t)] = \sigma\varepsilon[T_0^2 + T^2(d,t)][T_0 + T(d,t)][T_0 - T(d,t)] = h_r[T_0 - T(d,t)] \tag{5}$$

where $h_r = \sigma\varepsilon[T_0^2 + T^2(d,t)][T_0 + T(d,t)]$ is assumed to be approximately independent of temperature [8,9]. Further we let $T^*(z,t) = T(z,t) - T_0$ and consequently the boundary condition is not only linear but also homogeneous:

$$\frac{\partial T^*(d,t)}{\partial z} = (\frac{h_c + h_r}{k})\, T^*(d,t) = \beta T^*(d,t) \tag{6}$$

where $\beta = (h_c + h_r)/k$. Together with the boundary condition at the symmetric axis

$$\frac{\partial T(O,t)}{\partial z} = \frac{\partial T^*(O,t)}{\partial z} = O \tag{7}$$

and the initial conditions

$$T(z,0) = T_{sf} \qquad \text{and} \qquad T^*(z,0) = T_{sf} - T_0 \tag{8}$$

where T_{sf} is the stress-free temperature, we can solve Eq. (1) by the method of separation of variables.

The general solution takes the form

$$T^*(z,t) = e^{-\lambda^2\alpha t}(A_1 \sin\lambda z + A_2 \cos\lambda z) \tag{9}$$

Equation (7) yields $A_1 = 0$ and Eq. (6) gives

$$\lambda \tan\lambda d = \beta$$

This is a characteristic equation having an infinite number of roots — λ_n. Because the differential equation (Eq. (1)) is linear, any possible linear combination of the solutions is also a solution. The general solution then becomes

$$T^*(z,t) = \sum_{n=1}^{\infty} a_n e^{-\lambda_n^2\alpha t}\cos\lambda_n z \tag{10}$$

The remaining task is to determine a_n by the initial condition (Eq. (8)). This task can be done analytically only if $\cos\lambda_n z$ is an orthogonal series, which happens when either $\cos\lambda_n$ or $\sin\lambda_n$ is zero for all n (*i.e.* is $\lambda_n = n\pi$ or $\lambda_n = n\pi + \pi/2$) and the problem has the appropriate boundary conditions. Because β is positive, we apprehend that λ_n ♠ $(n-1)\pi$ for n=2, 3, 4, … and the larger the n, the closer they are. Therefore Eq. (10) can be approximately treated as an orthogonal series. By normalizing the half thickness of plate d to unity and using one of the criteria for the orthogonality condition in Ref. [8], we obtain

$$a_n = \frac{1}{b_n}\int_0^d (T_{sf} - T_0)\cos\lambda_n z\, dz \tag{11}$$

where

$$b_n = \int_0^d \cos^2\lambda_n z\, dz$$

Substitution of b_n into Eq. (11) gives

$$a_n = \frac{4(T_{sf} - T_0)\sin\lambda_n d}{2\lambda_n d + \sin 2\lambda_n d}$$

(12)

and finally, we have

$$T(z,t) = T_0 + (T_{sf} - T_0)\sum_{n=1}^{\infty}\frac{4\sin\lambda_n d}{2\lambda_n d + \sin 2\lambda_n d}\cos\lambda_n z\, e^{-\lambda_n^2 \alpha t}$$

(13)

Because the expression for a_n is approximate, we found it necessary to include more than 100 terms in Eq. (13) to get convergence to the correct answer.

For quenched laminates or laminates with a constant temperature surface, the boundary condition is simply

$$T^*(d,t) = 0$$

The general solution (Eq. (9)), symmetry condition (Eq. (7)) and initial condition (Eq. (8)) are still valid. The above boundary condition reveals

$$\cos\lambda d=0$$

which results in

$$\lambda_n=n\pi + \pi/2 \qquad n=1, 2, 3, ...$$

We can evaluate a_n by the same procedure used for convection and radiation cooling except that λ_n now defines an exact orthogonal series and the corresponding expression for a_n is therefore exact instead of approximate. The final expression of temperature distribution in quenched laminates is the same as Eq. (13) except the values of λ_n are changed.

PART TWO — THERMALLY-INDUCED RESIDUAL STRESSES

The material used in the present study is ICI PEEK/Hercules AS4 carbon fiber prepreg whose thermal expansion coefficient and Young's modulus were provided by ICI Composites. Due to the fiber dominant nature and the temperature insensitivity of the mechanical and thermal properties of carbon fibers, the mechanical and thermal properties in the fiber direction of composites can be assumed to be temperature independent. Experiments show that for this material only a 2.5% error will be introduced by using this assumption [10]. In contrast, the transverse mechanical and thermal properties are temperature dependent. ICI composites supplied experimental results for transverse mechanical and thermal properties from room temperature to the stress-free temperature.

In the analysis of thermally-induced residual stresses in a cross-ply laminate, so-called Classical Lamination Theory is used. Consider a flat plate of uniform thickness with an available temperature distribution T(z,t). Classical Lamination Theory gives [11]:

$$\{\sigma\} = [Q](\{\varepsilon\} - \{\alpha\}\Delta T(z,t)) \qquad (14)$$

$$\{\varepsilon\} = \{\varepsilon_0\} + z\{\kappa\}$$

where [Q] is the stiffness matrix, $\{\varepsilon_0\}$ are the strains in the mid-plane of the plate, $\{\alpha\}$ are the thermal expansion coefficients and $\{\kappa\}$ are the plate curvatures. Because we deal with symmetric laminates only, their curvatures due to temperature change are zero and therefore

$$\{\sigma\} = [Q](\{\varepsilon_0\} - \{\alpha\}\Delta T(z,t)) \qquad (15)$$

The residual stresses are zero at the stress-free temperature and start to build up as the laminate cools below this temperature. We assume that below the stress-free temperature the plate is solidified and the displacement along the thickness direction is uniform. Eq. (15) is a general expression for a temperature-independent material. Because the material we investigate is strongly temperature dependent, however, Eq. (15) has to be modified. If the temperature has an infinitesimal change from T to T+ΔT, the stresses change by

$$\{\Delta\sigma\} = [Q(T)]\{\Delta\varepsilon_0\} - [Q(T)]\{\alpha(T)\}\Delta T(z,t) \quad (16)$$

where [Q(T)] and {α(T)} are the stiffness matrix and thermal expansion coefficient at temperature T.

We slice the laminate into *m* thin layers and assume that each layer is thin enough to ignore the gradients of temperature as well as stress and strain. Because there are no applied forces, the equilibrium equation is taken to be

$$\int_D \sigma_p \, dz = 0$$

where p = x or y and D is the thickness domain. For a discretized thickness domain we have

$$\sum_{i=1}^{m} \Delta\sigma_i \Delta z_i = 0 \quad (17)$$

With the substitution of Eq. (16) into Eq. (17) and taking Δz_i as a constant we easily obtain

$$\sum[Q_{11i}(\Delta\varepsilon_x - \alpha_{xi}\Delta T) + Q_{12i}(\Delta\varepsilon_y - \alpha_{yi}\Delta T)] = 0$$

$$\sum[Q_{12i}(\Delta\varepsilon_x - \alpha_{xi}\Delta T) + Q_{22i}(\Delta\varepsilon_y - \alpha_{yi}\Delta T)] = 0$$

where Q_{11i}, Q_{12i}, α_{xi}, *etc*. are elements of the stiffness matrix and of the thermal expansion coefficient vector at temperature T; and $\Delta\varepsilon_x$ and $\Delta\varepsilon_y$ are variations of total strains in x and y directions, which are z independent, due to temperature variation from T to T + ΔT. These two equations lead to the expressions for $\Delta\varepsilon_x$ and $\Delta\varepsilon_y$

$$\Delta\varepsilon_x = \frac{(\Sigma Q_{11i}\alpha_{xi}\Delta T + \Sigma Q_{12i}\alpha_{yi}\Delta T)\Sigma Q_{22i} - (\Sigma Q_{12i}\alpha_{xi}\Delta T + \Sigma Q_{22i}\alpha_{yi}\Delta T)\Sigma Q_{12i}}{\Sigma Q_{11i}\,\Sigma Q_{22i} - (\Sigma Q_{12i})^2}$$

$$\Delta\varepsilon_y = \frac{(\Sigma Q_{12i}\alpha_{xi}\Delta T + \Sigma Q_{22i}\alpha_{yi}\Delta T)\Sigma Q_{11i} - (\Sigma Q_{11i}\alpha_{xi}\Delta T + \Sigma Q_{12i}\alpha_{yi}\Delta T)\Sigma Q_{12i}}{\Sigma Q_{11i}\,\Sigma Q_{22i} - (\Sigma Q_{12i})^2}$$

and eventually two expressions for the residual stress variation of each slice:

$$\Delta\sigma_{xi} = Q_{11i}(\Delta\varepsilon_x - \alpha_{xi}\Delta T) + Q_{12i}(\Delta\varepsilon_y - \alpha_{yi}\Delta T)$$

$$\Delta\sigma_{yi} = Q_{12i}(\Delta\varepsilon_x - \alpha_{xi}\Delta T) + Q_{22i}(\Delta\varepsilon_y - \alpha_{yi}\Delta T)$$

Finally the total residual stresses for each slice can be determined by summing each variation of stress in the temperature (or time) domain

$$\sigma_{xi} = \sum \Delta\sigma_{xi}$$

$$\sigma_{yi} = \sum \Delta\sigma_{yi}$$

NUMERICAL STUDY AND CONCLUSIONS

According to data provided by ICI Composites, we selected thermal conductivity k = 0.25 W/m-˚K, specific heat capacity C_p = 1.5 kJ/kg-˚K = 0.4167 W-hr/kg-˚K, and average density $\rho = 1.3$ g/cm^3. From other sources we selected the convective heat transfer coefficient $h_c = 2.5$ Btu/hr ft^2-°F = 14.186 W/m^2-°K [7], Stefan-Boltzmann constant $\sigma = 0.1714 \times 10^{-8}$ Btu/hr-ft^2-°R^4 =5.669x10^{-4} W/m^2-°K^4 [7], and surface emissivity $\varepsilon = 0.92$ [5].

Figures 1 and 2 illustrate the distributions of x-axis residual stresses for thin laminates $[0_2/90_2]_s$ and $[90_2/0_2]_s$. Figures 3 and 4 show the results for thick laminates $[0_{90}/90_5]_s$ and $[90_5/0_{90}]_s$ (about one inch thick). We draw the following conclusions:

(1) For thin laminates the slow cooling results are close to the convection and radiation results. This implies that the assumption of a uniform temperature distribution is adequate for thin laminates. For thick laminates the convection and radiation results are between those of slow cooling and quenching results, which indicates that an assumption of uniform temperature distribution is not adequate. An accurate estimation of the non-uniform residual stresses in thick laminates must use an analysis that accounts for heat conduction similar to the one in this paper.

(2) The residual stresses in quenched laminates, as well as in thick laminates under convective and radiative cooling, always have a high gradient at the laminate surface. The magnitude of stress variation in this area remains unchanged regardless of the laminate thickness. The high normal residual stress gradient caused by nonuniform cooling will likely produce a high shear stress gradient, which might cause local delamination.

(3) Beyond this high gradient stress area the residual stresses within each ply group remain nearly uniform. Thus, the residual stresses away from this area in thick laminates will be nearly unaffected by processing conditions.

COMMENTS

The processing conditions affect the cooling temperature distribution and may consequently cause nonuniform residual stresses. The effects of processing conditions are strongest near the surfaces of laminate where the residual stresses can differ significantly from those calculated by a simple laminated plate theory that assumes a uniform temperature distribution during cooling. At the center of laminates, a simple uniform temperature distribution gives a good estimate of the ply residual stresses.

Numerical results support the claim that the residual stresses in "quenched" laminates and in slowly cooled laminates provide upper and lower bounds to the residual stresses. Either one may provide the upper bound somewhere and the lower bound somewhere else. The results for convection and radiation, however, will definitely be bounded by the upper and lower bounds. In real applications, it will be very hard, if not impossible, to precisely describe the cooling boundary conditions in processing. If the upper and lower bounds are given, it will help the designer to have an estimate on the level of the residual stresses.

We believe that below the stress-free temperature of PEEK composites that the plate is solidified and is also fully crystalized [6]. Any significant crystallization happening below the stress-free temperature might cause volume reduction and result in extra residual stresses. We also assumed that viscoelastic behavior of PEEK material does not significantly influence the residual stresses. If a great amount of time is spent above the glass transition temperature, it is possible that stress-relaxation will reduce the level of residual stresses. Most residual stresses form, however, when the matrix is stiff and below the glass transition temperature. At these lower temperatures, stress relaxation effects are probably minimal.

ACKNOWLEDGMENTS

This work was supported in part by a contract from NASA Langley Research Center (NAS1-18833) monitored by Dr. John Crews, in part by a gift from ICI Advanced Composites monitored by Dr. J. A. Barnes, and in part by a gift from the Fibers Department of E. I. duPont deNemours & Company monitored by Dr. Alan R. Wedgewood.

REFERENCES

1. Jeronimidis, G., and Parkyn, A. T., “Residual Stresses in Carbon Fiber-Thermoplastic Matrix Laminates,” Journal of Composite Materials, Vol. 22, 1988, pp. 401-415.
2. Hahn, H. T., “Residual Stresses in Polymer Matrix Composite Laminates,” Journal of Composite Materials, Vol. 10, 1976, pp. 266-278.
3. Hussein, R. et al, “Thermal Stresses in Sandwich Plates,” Journal of Thermal Stresses, Vol. 12, 1989, pp. 333-349.
4. Griffis, C. A., et al, “Degradation in Strength of Laminated Composites Subjected to Intense Heating and Mechanical Loading,” Journal of Composite Materials, Vol. 20, 1986, pp. 216-220.
5. Chen, J. K., et al, “Failure Analysis of a Graphite/Epoxy Laminate Subjected to Combined Thermal and Mechanical Loading,” Journal of Composite Materials, Vol. 19, 1985, pp. 408-416.
6. Nairn, J. A., and Zoller, P., “Residual Thermal Stresses in Semicrystalline Thermoplastic Matrix Composites,” Fifth International Conference on Composite Materials (ICCM-V) July, 1985.
7. Sucec, J., Heat Transfer, WM. C. BROWN Publishers, Dubuque, Iowa, 1985.
8. Mayers, G. E., Analytical Methods in Conduction Heat Transfer, McGraw-Hill Book Company, New York, 1971.
9. Biot, M. A., Variational Principles in Heat Transfer, Oxford University Press, London, 1970.
10. J. A. Barnes, I. J. Simms, G. J. Jackson, D. Jackson, G. Wostenholm, and B. Yates, “Thermal Expansion Behaviour of Thermoplastic Composites,” ASME Winter Meeting, Dallas, Texas, November 25-30, 1990.
11. Tauchert, T. R., “Thermal Stresses in Plates — Statical Problem,” Chapter 2 of Thermal Stresses I, R. B Hetnarski Ed., North-Holland, Amsterdam, 1986.

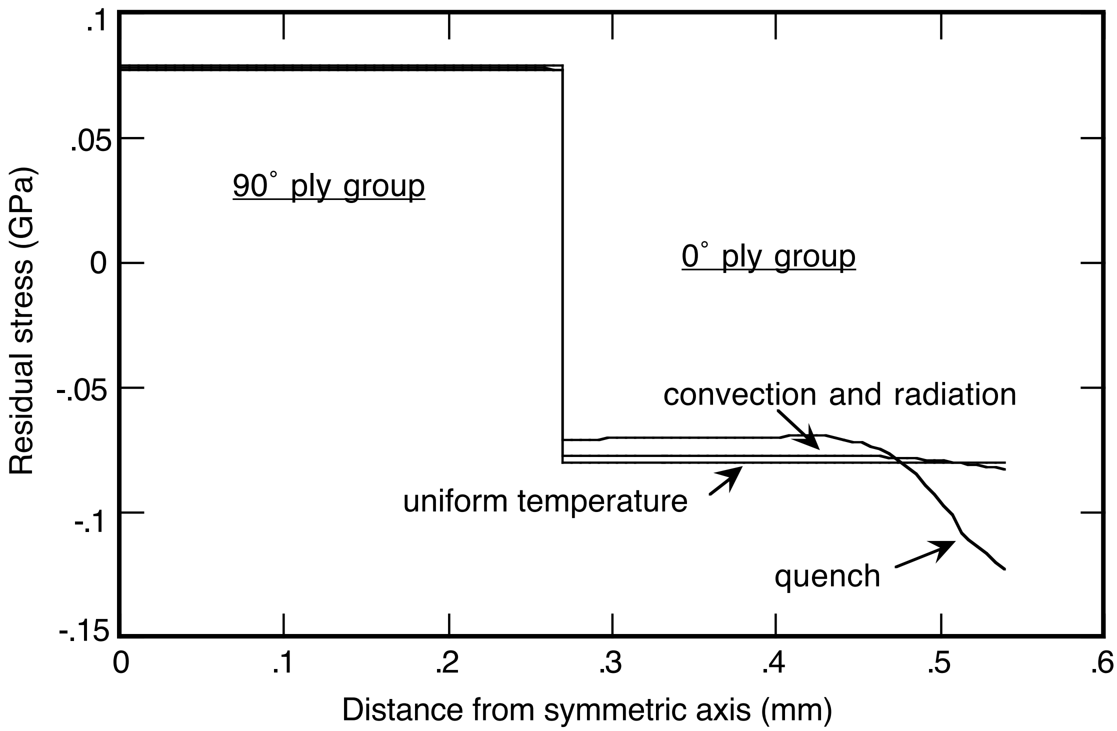

Figure 1: The distributions of x-axis residual stresses in a $[0_2/90_2]_s$ laminate cooled under uniform temperature distribution (slow cooling), by convection and radiation, and by quenching to room temperature.

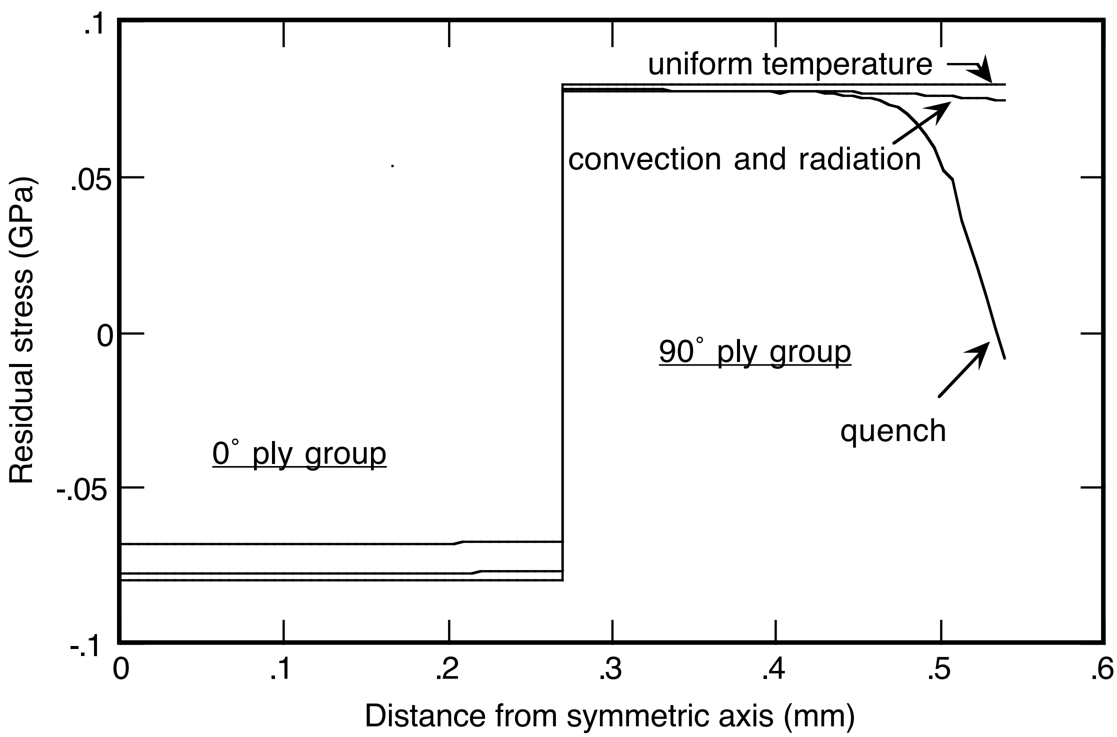

Figure 2: The distributions of x-axis residual stresses in a $[90_2/0_2]_s$ laminate cooled under uniform temperature distribution (slow cooling), by convection and radiation, and by quenching to room temperature.

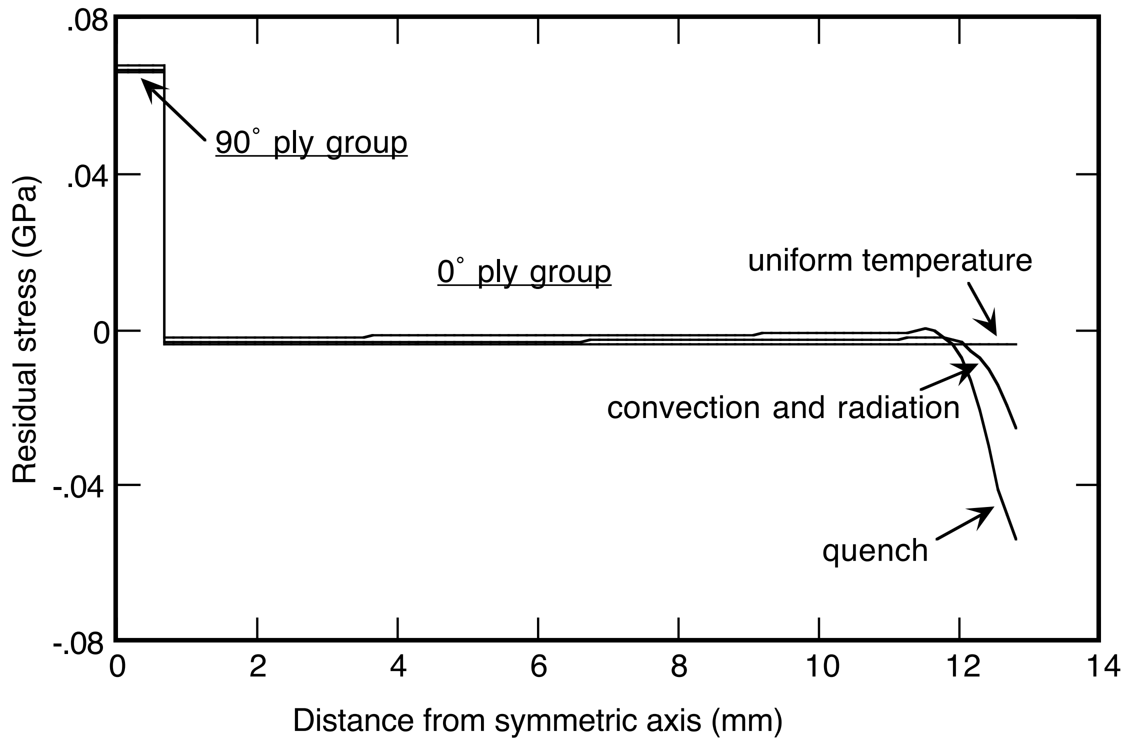

Figure 3: The distributions of x-axis residual stresses in a $[0_{90}/90_5]_s$ laminate cooled under uniform temperature distribution (slow cooling), by convection and radiation, and by quenching to room temperature.

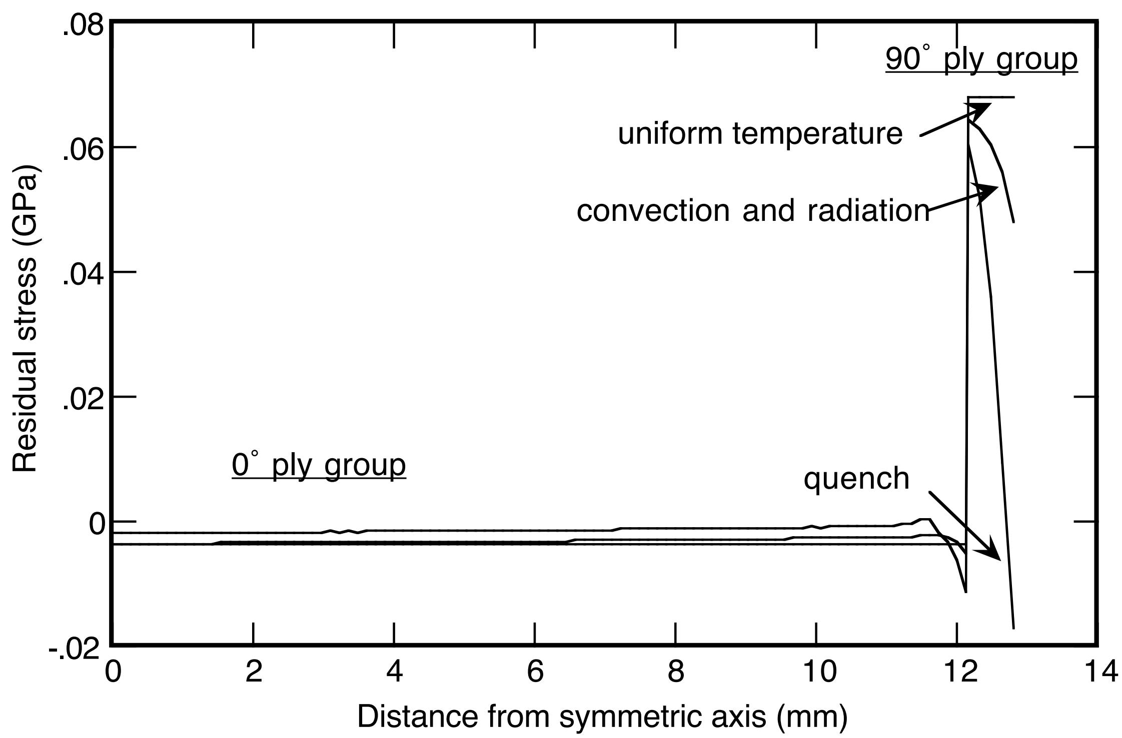

Figure 4: The distributions of x-axis residual stresses in a $[90_5/0_{90}]_s$ laminate cooled under uniform temperature distribution (slow cooling), by convection and radiation, and by quenching to room temperature