

RESIDUAL STRESS EFFECTS IN FRACTURE OF COMPOSITES AND ADHESIVES

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ABSTRACT

Because composites and adhesive joints are made from different phases with different thermal expansion coefficients, they inevitably develop residual thermal stresses. When designing composites or adhesive joints, it is important to account for these residual stresses. It is not the magnitude of the residual stresses that matters, but rather the effect of those stresses on composite failure properties. This effect can be assessed by including residual stresses in fracture mechanics models of failure. This chapter gives general results for energy release rate in the presence of residual stresses. In many important problems, the effect of residual stress can be evaluated without any need for thermoelasticity analysis of the structure. Several examples of including residual stresses in fracture mechanics interpretation of experimental results are discussed.

KEYWORDS

Keywords: Residual Stresses, Thermal Stresses, Adhesive Joints, Microcracking, Coatings.

INTRODUCTION

Composites and structures, such as adhesive joints, are comprised of multiple phases having different thermal expansion coefficients. Because manufacturing methods typically include elevated temperature steps, when the final structure is cooled to use temperature, residual stresses occur. Residual stresses are not necessarily a problem; in some situations they can be beneficial. In other situations, however, residual stresses have an adverse effect on failure properties. Residual stresses may cause a composite to fail sooner than expected, to have greater susceptibility to solvents, or to have reduced durability due to accelerated aging or fatigue damage mechanisms. Thus an important question in all design and analysis with composite materials and adhesive joints is how do residual stresses affect failure properties?

This chapter considers fracture mechanics analysis of composites and adhesive joints with residual thermal stresses. The methods here are restricted to structures with linear thermoelastic phases subjected to a uniform change in temperature. Application of fracture mechanics requires evaluation of the energy release rate, G , for propagation of cracks in the presence of residual stresses. In many important problems, the effect of residual stresses on G can be evaluated without the need for any thermoelasticity analysis for residual stresses. This general result makes it possible to easily include residual stress effects in models that previously incorrectly ignored them. The general theory for fracture of structures with residual stresses is given in the theory section. The results section gives some examples of residual stress effects on failure in composites and adhesive joints.

THEORY OF RESIDUAL STRESS EFFECTS

Consider an arbitrary structure subjected to a uniform temperature change of ΔT and to any mixed traction and displacement boundary conditions as illustrated in Fig. 1. This structure may contain cracks within phases or cracks spanning phases. The goal of any fracture mechanics analysis is to derive the energy release rate for crack growth, which can be calculated from global thermoelastic potential energy using

$$G = -\frac{d\Pi}{dA} = \frac{d(W - U)}{dA} \quad (1)$$

where Π is the thermoelastic potential energy, W is external work, U is thermoelastic internal energy, and A is crack surface area. By partitioning total stresses into mechanical and residual stresses (σ^m and σ^r) and making use of virtual work methods and the divergence theorem, G can be expressed in a useful and general form as (see Refs. [1] and [2] for details):

$$G = G_{mech} + \frac{V\Delta T}{2} \left(2 \frac{d\langle \sigma^m \cdot \alpha \rangle}{dA} + \frac{d\langle \sigma^r \cdot \alpha \rangle}{dA} \right) \quad (2)$$

where V is total volume and G_{mech} is the energy release rate in the absence of residual stresses:

$$G_{mech} = \frac{d}{dA} \left(\frac{1}{2} \int_{S_r} T^0 \cdot u^m dS - \frac{1}{2} \int_{S_u} T^m \cdot u^0 dS \right) \quad (3)$$

The angle brackets indicate volume-averaged quantities; here they are average mechanical or residual stresses weighted by the phase-dependent thermal expansion tensor α . T and u in Eq. (3) are surface tractions and displacements; superscript m indicates the mechanical component of the boundary terms. These results assume traction free crack surfaces and perfect interfaces. The extension of traction loaded cracks and imperfect interfaces is given in Ref. [2].

The first term in Eq. (2) is G in the absence of residual stresses; thus the other terms are an exact expression of the residual stress effect in composite fracture. An important special case of Eq. (2) is to consider mode I fracture. During pure mode I crack growth (and similarly during pure mode II or mode III crack growth) the mode I energy release rate must be proportional to mode I stress intensity factor squared, K_I^2 . Furthermore, for linear-elastic materials in which all applied tractions and displacements are scaled by a factor P and the temperature difference ΔT

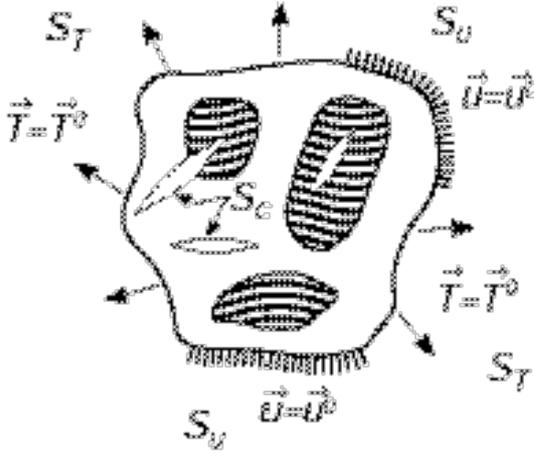


Fig. 1: An arbitrary multiphase composite subjected to tractions \mathbf{T}^0 on surface S_T and displacement boundary conditions \mathbf{u}^0 on surface S_u and containing crack surfaces (S_c).

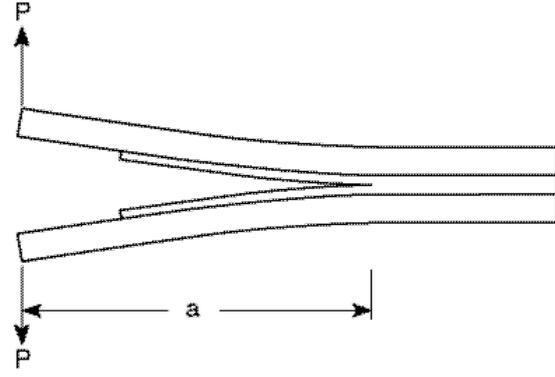


Fig. 2: A DCB specimen used to measure mode I toughness of adhesive joints. P is the applied load and a is crack length.

is scaled by a factor T^* , K_I must scale by a linear combination of P and T^* . In other words, the mode I energy release rate must have the form

$$G_I \propto K_I^2 = (c_1 P + c_2 T^*)^2 \quad (4)$$

where c_1 and c_2 are constants that depend on the specific problem. Comparing Eq. (4) to Eq. (2) and realizing that σ^m is proportional to P while σ^r is proportional to T^* , it is possible to eliminate one term and simplify G_I to [1,2]

$$G_I = G_{mech} \left(1 + \frac{V \Delta T}{2G_{mech}} \frac{d\langle \sigma^m \cdot \alpha \rangle}{dA} \right)^2 \quad (5)$$

Notice that G_I depends only on mechanical stresses. Thus once the mode I fracture problem is solved ignoring residual stresses, the energy release rate in the presence of residual stresses can be determined exactly without any need for thermoelasticity analysis of the structure.

The general result in Eq. (5) has many applications for including residual stress effects in the analysis of fracture of composites and structures. It is analogous to the well-known Levin [3] analysis that derived a method to exactly calculate effective thermal expansion coefficient of a composite from knowledge of mechanical stresses alone. The Levin analysis used virtual work methods and did not need any thermoelasticity results. Similarly, Eq. (5) was derived with virtual work methods and only needs a mechanical stress analysis. Both analyses are limited to linear-elastic materials; Eq. (5) is further limited to pure mode fracture (pure mode I, II, or III), but many important failure problems are pure mode fracture problems.

RESIDUAL STRESS EFFECT RESULTS

Figure 2 shows a double cantilever beam specimen (DCB) used to measure the fracture toughness of adhesive joints [4]. Assuming the crack runs down the middle of the adhesive as pure mode I fracture, the energy release rate in the presence of residual stresses can be evaluated using Eq. (5) and corrected beam theory to be [5]

$$G_I = (C_m (a + 1.15\Delta_w)P + C_r \Delta T)^2 \quad (6)$$

where the mechanical and residual constants are given by

$$C_m = \sqrt{\frac{C_{\kappa}^*}{B}} \quad \text{and} \quad C_r = \frac{\alpha_{\kappa}^*}{\sqrt{C_{\kappa}^* B}} \quad (7)$$

Here C_{κ}^* and α_{κ}^* are the simple beam theory compliance and thermal curvature coefficient of one arm of specimen [5], and B is width of the arms. The mechanical term includes an *effective* crack length $(a + 1.15\Delta_w)$ as discussed elsewhere for corrected beam theory of DCB specimens [5,6]; the coefficient 1.15 on the correction term Δ_w [6] was determined numerically by finite element analysis of numerous DCB specimens [5]. The thermal term does not require correction because it is accurate with simple beam theory alone [5]. Comparison to finite element analysis shows that Eq. (6) is accurate within 1% for all adhesive DCB geometries [5]. It is thus easy to correct adhesive tests for residual stress effects. Notice that the residual stress effect in G_I is proportional to α_{κ}^* , which means the residual stress effect is caused by a tendency of the arms of the specimen to curve due to residual stresses.

Many fracture studies on adhesives have ignored residual stresses [4]. Equation (6) can be used to calculate the errors of such an approach [5]. Assume that some particular adhesive has a *true* fracture toughness of G_{Ic} . If an adhesive DCB specimen with this adhesive is tested, it will fail when $G_I = G_{Ic}$, which, by Eq. (6), happens when the load reaches

$$P = \frac{\sqrt{G_{Ic}} - C_r \Delta T}{C_m (a + 1.15\Delta_w)} \quad (8)$$

If this observed failure load is then used in an analysis that ignores residual stresses (*i.e.*, Eq. (6) with $\Delta T=0$), the calculated or apparent toughness is

$$G_{Ic}^{app} = (\sqrt{G_{Ic}} - C_r \Delta T)^2 \quad (9)$$

The difference between G_{Ic} and G_{Ic}^{app} is the error caused by ignoring residual stresses. In typical polymer adhesives between metal adherends, $C_r \Delta T$ is negative which causes G_{Ic}^{app} to be *higher* than the actual toughness. In other words, the presence of residual stresses causes the adhesive to appear tougher than it does without residual stresses. Some sample calculations of errors due to ignoring residual stresses are plotted in Fig. 3 as the percentage error in G_{Ic}^{app} as a function of modulus ratio between adherend and adhesive, R , for various adherend to adhesive thickness ratios, λ [5]. The errors are large for low R and λ and decrease as either R or λ increases. The dashed vertical line shows a typical R value for aluminum-epoxy specimens. The aluminum-epoxy errors exceed 1% even with a very thin adhesive ($\lambda=64$) and exceed 40% for a thick adhesive ($\lambda=2$).

A similar analysis [5] can be done for laminate double cantilever beam specimens used to measure delamination toughness. The residual stress effect in DCB specimens can be eliminated by constructing doubly-symmetric double cantilever beam specimens in which each arm of the specimen is a symmetric laminate and thus has $\alpha_{\kappa}^* = 0$. If the arms are not symmetric laminates, however, the residual stress effect can be very large. For example, the errors in G_{Ic}^{app}

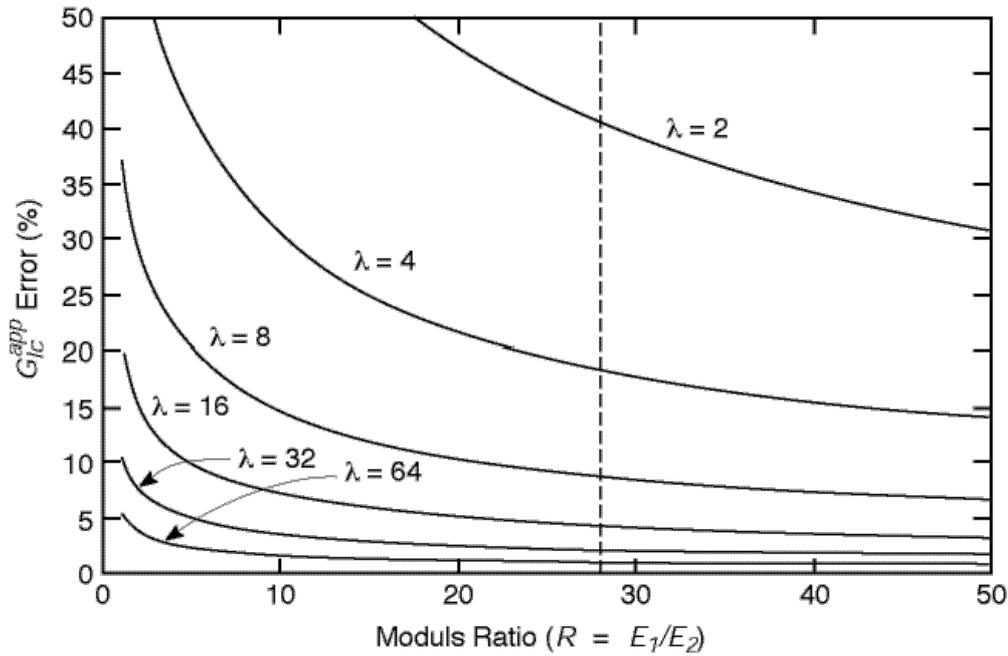


Fig. 3: The percentage error in the apparent toughness G_{Ic}^{app} in adhesive DCB specimens as a function of adherend to adhesive modulus ratio, R , for various values of adherend to adhesive thickness, λ . The calculations assumed $G_{Ic}=200 \text{ J/m}^2$, $\Delta\alpha = -40 \times 10^{-6} \text{ K}^{-1}$, and $\Delta T = -100^\circ\text{C}$. The vertical dashed line corresponds to $R=28$, which is a typical value for aluminum-epoxy specimens.

for a variety of laminates were calculated to range from -55% to +76% [5]. In other words, the errors caused by ignoring residual stresses can be large and can cause the G_{Ic}^{app} result to be either significantly too high or significantly too low.

Additional examples of residual stress effects in fracture of composite or structures include matrix microcracking in laminates [7,8], cracking of paints or coatings [9], and interfacial crack growth such as in the microbond or pull-out specimens [10-12]. In matrix microcracking of cross-ply laminates, the residual stress effect induces tensile stresses in the 90° plies that promote microcracking. The general methods above can include residual stresses in the energy release rate for microcracking; proper analysis of microcracking experiments requires inclusion of the residual stress term. A master plot method is available which make it possible to determine both toughness and residual stress effects from experimental cracking experiments [7,8]. In other words, the value of ΔT in the analysis does not have to be measured or be assumed to be due to linear thermoelastic phases; it can be determined from fracture experiments as an effective ΔT for residual stresses. Cracking of coatings is analogous to microcracking of laminates [9]. An additional source of residual stresses in coatings might arise from chemical shrinkage as the coating cures or dries. Such residual stresses can be included in a thermal stress analysis by replacing thermal strain terms like $\Delta\alpha\Delta T$ by an effective residual strain term due to both chemical and thermal residual stresses [9].

In the microbond specimen [13], a droplet of matrix is deposited on a fiber and an interfacial crack is induced by pulling the fiber while restraining the matrix. In this specimen, all loading is on the top of the droplet while the bottom of the droplet is stress free. A global energy analysis shows that the bulk of the energy released due to crack growth is due to the release of

residual stress energy as the droplet and matrix become debonded [10]. Although many interpretations of microbond results have ignored residual stresses, such models have serious errors. By including residual stresses and interfacial friction effects, the microbond specimen can become a fracture mechanics test for determining interfacial mode II fracture toughness [10,14]. For example, it can be used to assess the role of physical aging and residual stress relaxation on interfacial properties [11]. Similar fracture mechanics methods can be used to include residual stress effects in the analysis of single-fiber, pull-out experiments [10,12].

RECOMMENDATIONS

Residual stresses are always present in composites and structures such as adhesive joints. Proper analysis of fracture experiments with such materials must therefore always include residual stress effects. In some geometries, such as tapered DCB specimens with very large R and λ , residual stresses can be demonstrated to be small and can be ignored [15]. In many important specimens, however, residual stresses are non-negligible and can even be the dominant effect causing fracture. The methods above give general tools for including residual stresses. These tools assume linear thermoelastic phases with temperature-independent properties. The general equations can be extended to more general phases and to other sources of residual stresses by replacing ΔT by an effective value that gives the actual level of residual stresses. The use of an effective ΔT usually means the fracture experiments must be coupled with additional experiments that measure the level of residual stresses in the specimen.

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