NUMERICAL MODELING OF DEFORMATION AND FRACTURE OF WOOD INCLUDING HETEROGENEITY AND ANISOTROPY

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Abstract
The challenge in numerical modeling of wood is to have the model closely match the structure of a real specimen. The model should account for heterogeneity, such as wood grain, growth rings, etc., and for anisotropy of each phase. In some problems, the model should also match realistic morphologies of specimens. This paper presents numerical results for transverse modulus and for transverse fracture using methods that explicitly account for earlywood and latewood and account for anisotropy in the radial and tangential directions. The fracture results analyzed a realistic morphology using digitized image of an actual specimen. A companion, imaged representation of radial and tangential directions was used to include anisotropy in the transverse plane. Experimental and simulation results on the same specimen can be compared. Such coupled methods may shed new light on deformation and fracture properties of solid wood.

1 INTRODUCTION

Wood is a cylindrically orthotropic material with its longitudinal direction along the tree's axis and its radial and tangential directions normal and tangential to the growth rings in a transverse cross section of the tree. Lumber is typically sawn from trees with rectangular cross sections. The arrangement of the polar material within rectangular lumber influences the properties of the lumber.

Four common methods for numerical modeling of wood on problems that include the transverse plane are:

- Transversely isotropic material: The simplest model is to assume wood is isotropic in the transverse plane. The justification for his approach is that the radial and tangential moduli are similar. Because an isotropic plane has shear modulus about 1/3 the tensile modulus while the transverse shear modulus of wood, G_{RT}, is 10-20 lower than transverse tensile moduli, this model is poor for properties that are influenced by wood's low shear modulus.
- Rectilinear orthotropic material: This model ignores curvature of growth rings by assuming the longitudinal, radial, and tangential directions align with the x, y, and z directions of rectangular lumber. This model is poor for effects due to the polar structure of wood.
- Homogenized cylindrical orthotropy: This model accounts for wood's polar structure, but simplifies the analysis by homogenizing properties of earlywood and latewood into bulk properties. This model can account for the polar structure of wood, but can not account for effects due to differential properties between earlywood and latewood.
- Heterogeneous cylindrical orthotropy: The most realistic models explicit account for the polar structure of wood and variations in material properties between earlywood and latewood. This approach is the most challenging numerically and hindered by lack of reliable material properties for earlywood and latewood layers.

This paper presents two examples of numerical modeling of wood using the heterogeneous cylindrical orthotropy approach. Some of the results required this level of modeling thus illustrating that common approximations used in prior numerical models may miss important aspects in the physics of deformation and fracture of solid wood.

2 EXAMPLES

2.1 Transverse Modulus

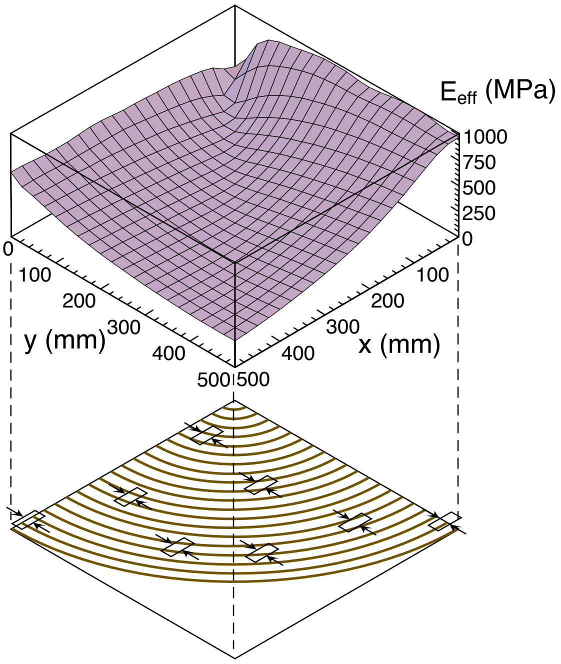

Fig. 1 Transverse modulus of Douglas Fir for all possible end-grain patterns plotted as a function of board centroid location in on quadrant of a tree.

The radial and tangential moduli (E_R and E_T) of wood have some unusual experimental properties. Bodig [1] observed that E_R of Douglas fir increases as specimen thickness increases. Kennedy [2] measured transverse modulus as a function of loading angle for several species and found that the modulus varies with angle and can even be lower than both E_R and E_T. Shipsha and Berglund [3] found that E_R increases with distance of the board from the pith. These observations are a consequence of the transverse anisotropy of wood and of fitting a cylindrical material into a rectangular specimen. Previous finite element analysis (FEA) confirm that transverse modulus is strongly affected by the low transverse shear modulus, G_{RT}, of wood [3,4], but these results used the homogenized cylindrical orthotropy model. This sections gives new results using the heterogeneous cylindrical orthotropy model.

Figure 1 summarizes the results for FEA calculations of the transverse modulus of Douglas fir boards with 441 different end-grain patterns. In brief, all end-grain patterns can be sampled by selecting board centroid within one quadrant of a tree. The plot gives modulus as function of board centroid. For radial loading (along $x = 0$), the modulus was low near the pith, but increased to the bulk E_R far from the pith. Pure tangential loading (along $y = 0$) was similar, except with a smaller increase because the bulk E_T is smaller. For orientations deviating from pure radial or tangential loading, the modulus rapidly decreased due to wood's low G_{RT}. From the FEA results, the effective transverse modulus was lower than both E_R and E_T for 82% of the 441 orientations.

This heterogeneous modeling required input of separate earlywood and latewood properties. The assumed properties are in Table 1. These properties were not measured, but they are consistent with all experimental results on earlywood and latewood, with theories of cellular materials, and with measured bulk properties [5]. The numerical results are qualitatively similar to FEA results using homogenized properties, but some details are sensitive to separate earlywood and latewood properties or the layered structure of wood. Most dramatically, stress and strain concentrations are much greater in a heterogeneous model then in a homogenized model [5].

Table 1: Properties for bulk wood, earlywood, and latewood used in the calculations to model Douglas fir in the longitudinal (*L*), radial (*R*), and tangential (*T*) directions.

Property	Bulk	Earlywood	Latewood
E_T (MPa)	620	152	1215
E_R (MPa)	960	566	1752
E_L (MPa)	14500	10400	20700
G_{RT} (MPa)	80	50	215
ν_{TR}	0.35	0.30	0.425
ν_{TL}	0.033	0.033	0.033
ν_{RL}	0.041	0.041	0.041
fraction	—	0.60	0.40

2.2 Transverse Fracture

Two challenges to modeling using the heterogeneous cylindrical orthotropy approach are modeling a realistic morphology of wood and modeling orientation of material properties that vary throughout the specimen. These challenges recommend a new method called the material point method (MPM) [6,7]. MPM is a particle based method.

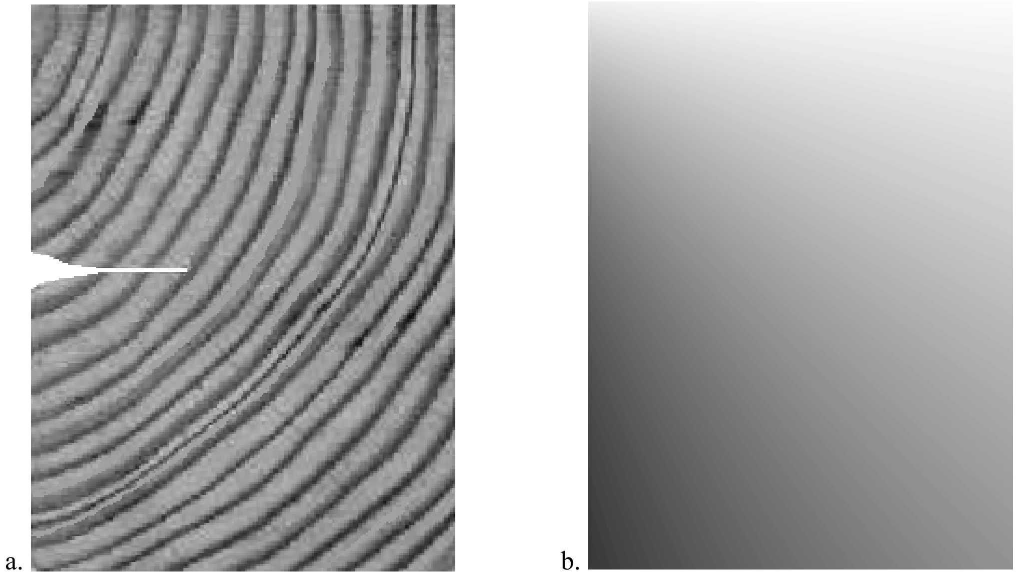

Fig 2 a. Digitized image of Douglas fir with a notch. b. Estimated image for angle of radial direction from the x axis on scale of 0° to 90° from white to black.

As such, one approach to modeling realistic morphologies is convert pixels in an image into particles in an analysis. The properties of the particles can be assigned to material properties based on color or intensity of the the image. For example, Fig. 2a shows a digitized image of Douglas Fir with a crack. This image can be directly converted to an MPM model by converting dark pixels to latewood and light pixels to early wood. A sample MPM model (after simulating crack growth) is shown in Fig 3b.

The intensities of an image define earlywood and latewood, but they do not provide information about the radial and tangential directions of each particle. A new method has been developed to couple the first image to a second image that gives orientation information. For example, Fig 2b shows an intensity map where the intensity defines orientation of the radial direction as clockwise angle from the x axis. The mapping is from white to black for radial direction from 0° to 90°. In summary, two images are input to the MPM model. The first image defines morphology of earlywood and latewood; the second image defines orientation of each particle. Going directly from two images for a complete heterogeneous cylindrical orthotropy model of wood is an advantage of MPM. The orientation image here was approximated by constructing a radial blend in graphics software. A preferred approach, in development, would be to develop image analysis filters than can calculate orientation information from the original image in Fig. 2a [8].

The next step was to do a fracture experiment on the actual specimen and compare the results to numerical modeling on the same wood morphology. The numerical modeling of fracture is very complex. The model needs to make numerous assumptions about fracture and mechanical properties of earlywood and latewood, about critical conditions for crack growth, and once a crack grows, a scheme for predicting the crack growth direction. These simulations used the mechanical properties in Table 1 and assumed the toughness of earlywood and latewood are the same. The crack growth direction was based on an estimation of the direction for maximum energy release rate from the normal and shear crack opening displacements. The goal was to assess the role of heterogeneity in mechanical properties on crack propagation in wood.

One sample calculation in shown in Fig. 3. Figure 3a shows the experimental results; Fig. 3b shows the MPM simulation. The simulations match many details of the experiments. In particular, the crack tends to turn in the radial direction when in latewood, but to veer away from the latewood when in earlywood. The result is an oscillating crack path. Numerous simulations were run for other specimens and for various orientations of the crack with respect to growth rings. Some observations are:

- The crack direction oscillates as it moves between earlywood and latewood. The magnitude of the oscillations increases as G_{RT} of the earlywood decreases (in simulations with lower G_{RT} in

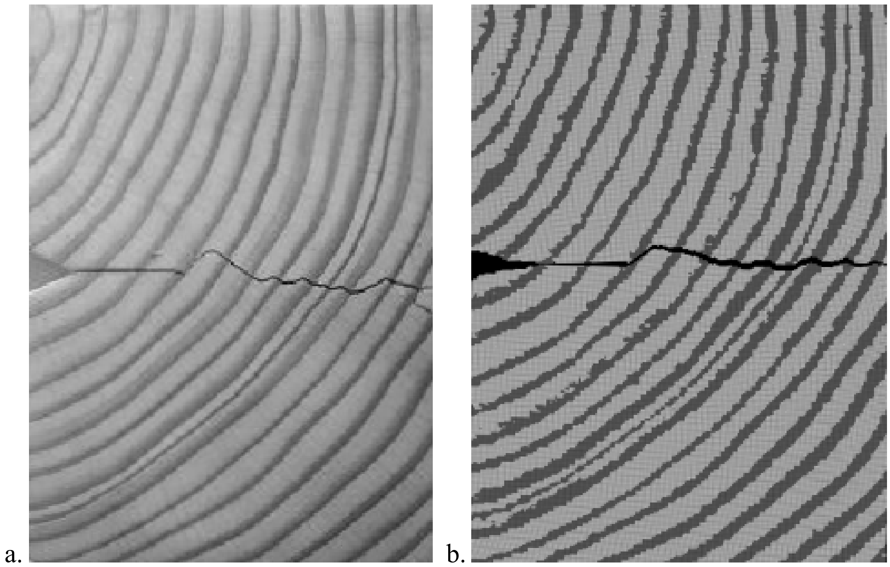

Fig. 3 a. Experimental results for transverse crack growth in Douglas fir. The initial specimen is in Fig. 2a. b. MPM results for simulated crack growth. The crack was started with an initial kink. The turing of the crack back to the horizontal direction and oscillations through earlywood and latewood are simulations results.

earlywood, the G_{RT} of latewood was increased to keep the bulk G_{RT} equal to bulk G_{RT} for Douglas fir).

- Simulations using homogenized properties lack all oscillations and thus homogenized methods can not simulate the details of transverse fracture.
- Some direction shifts are more abrupt in real specimens than in simulation results. For example the initial crack growth in Fig. 3a turns in the tangential direction. Simulations could not reproduce this abrupt change. Therefore the simulation in Fig 3b was started from an initially kinked crack and then looked at subsequent crack growth after the kink. Such abrupt changes in direction may be a consequence of heterogeneity in toughness or of flaws in the material. For example, sample preparation may have created a flaw that caused the initial kink while the subsequent *natural* crack propagation was closer to the simulation. These simulations looked only at mechanical property variation effects and not at toughness variation effects.

3 REFERENCES

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