MATERIAL POINT METHOD CALCULATIONS WITH EXPLICIT CRACKS, FRACTURE PARAMETERS, AND CRACK PROPAGATION

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ABSTRACT

A new particle-based method, the material point method (MPM), has recently been extended to handle explicit cracks in a new algorithm called CRAMP or "CRAcks with Material Points." This new method has several advantages for numerical work on fracture. Compared to finite element analysis, CRAMP can handle cracks with similar algorithmic efficiency, but is better at handling crack surface contact and crack propagation in arbitrary directions. Compared to meshless methods, CRAMP can handle arbitrary crack propagation with similar ease, but is better at inclusion of explicit cracks. MPM/CRAMP also works well for calculating key fracture parameters such as J integral, stress intensity factors, or crack-opening displacements. This extended abstract summarizes the approach of the MPM/CRAMP method and illustrates it with several example calculations including crack propagation.

1 INTRODUCTION

Numerical modeling has always been an integral part of fracture characterization of materials. For example, all standards for plane-strain fracture toughness testing rely on numerically evaluated geometric factors for determination of toughness or critical stress intensity factors. Modern numerical fracture work is done using various numerical methods. This extended abstract discusses a new numerical method, called the material point method (MPM), to provide more options for fracture modeling. MPM is a particle-based or meshless method that uses a background grid as a computational scratch pad. An advantage of the background grid is that MPM can handle explicit cracks with the accuracy and efficiency of finite element analysis (FEA). Because the solution is particle based, however, MPM can also handle crack propagation with the ease of meshless methods. This combination of features recommends MPM as a valuable tool for studying fracture, especially for problems involving crack propagation.

2 MATERIAL POINT METHOD WITH CRACKS

The material point method (MPM) has recently been developed as a numerical method for solving problems in dynamic solid mechanics (Sulsky [1,2]). In MPM, a solid body is discretized into a collection of material points or particles much like a computer image is represented by pixels. As the dynamic analysis proceeds, the solution is tracked on the particles by updating all required properties such as position, velocity, acceleration, stress state, temperature, *etc.*. At each time step, the particle information is extrapolated to a background grid that serves as a computational scratch pad for solving the equations of motion. Once the equations are solved, the grid-based solution is used to update all particle properties. This combination of particle basis (Lagrangian) and non-body-fixed grid methods (Eulerian) has proved useful for solving problems with large deformations or rotations, with materials having history dependent properties (such as plastic or viscoelastic materials – Sulsky [1,2]), or with complicated geometries (such as foams or granular materials – Bardenhagen [3,4]).

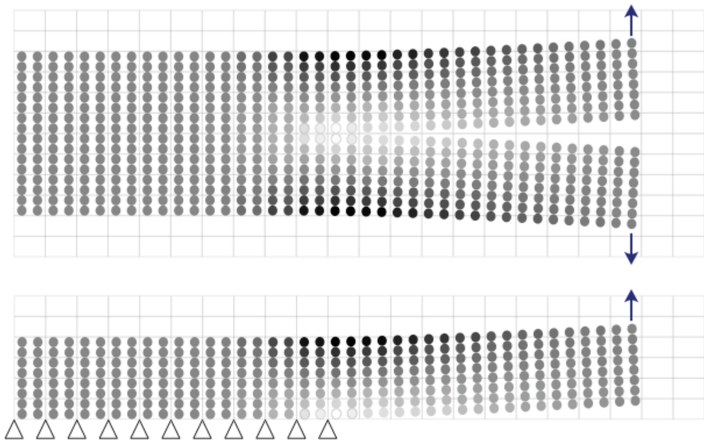

Figure 1: Analysis of a DCB specimen with an explicit crack using CRAMP (top) compared to analysis by symmetry using conventional MPM (bottom). Shades of gray (black to white) indicate tensile stress in horizontal direction.

Although MPM uses a background grid, a recent generalization of MPM (Bardenhagen [5]) reveals it is a Petrov-Galerkin method that has more similarities with meshless methods, such as Element-Free Galerkin (EFG) methods (Belytschko [6]) and Meshless-Local Petrov-Galerkin (MLPG) methods (Atluri [7]), then it does with FEA methods. The "meshless" aspect of MPM arises because the body and the solution are described by the particle states while the grid is solely a computational scratch pad (non-body-fixed grid). Thus, MPM has advantages of meshless methods such as ease in describing crack propagation in arbitrary directions and elimination of mesh distortion. The availability of a grid, however, provides some advantages in the area of computational efficiency and accuracy. For example, the grid makes it possible to handle explicit cracks (Nairn [8]) better than other meshless methods and simplifies calculation of fracture parameters such as J integral (Guo [9]).

MPM, as initially derived (Sulsky [1,2]), was not capable of handling explicit cracks. The problem is that conventional MPM extrapolates particle information to a single velocity field on the grid. A property of the grid, which is analogous to FEA meshes and many meshless interpolation schemes, is that displacements and velocities are continuous. Because cracks are displacement and velocity discontinuities, conventional MPM cannot represent cracks. We have recently derived a modified MPM labeled as CRAMP for "CRAcks" with "Material Points" that extends MPM to handle explicit cracks (Nairn [8]). The modification in CRAMP was to allow each node on the background grid to have multiple velocity fields. For nodes near cracks, there will be two velocity fields corresponding to extrapolated information from opposite sides of the crack. Cracks surfaces are discretized into a collection of massless particles (the particles are connected by line segments in 2D or triangular surfaces in 3D). The main modifications in CRAMP are to determine the appropriate velocity field for each particle/node pair, to solve equations of motion separately for each velocity field, and to update particle information using the appropriate velocity field depending on the location of the particle relative to cracks. The appropriate velocity field is determined by calculating whether or not a line *from* each particle *to* each node crosses a crack. CRAMP also tracks motion of crack surfaces that are used when

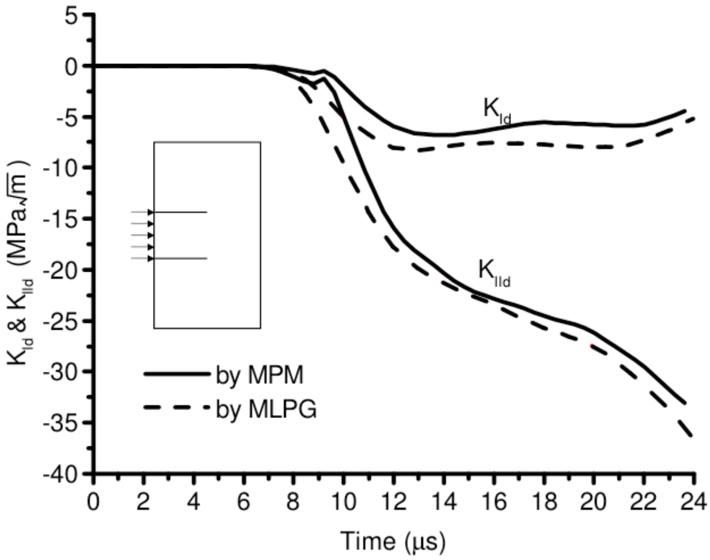

Figure 2: $K_I(t)$ and $K_{II}(t)$ dynamic stress intensity factors for DENP specimen calculated by CRAMP or by MLPG.

implementing crack surface contact laws by stick or by sliding with friction (Nairn [8]), and are used to calculate fracture parameters (Guo [9]).

2.1 Crack Patch Test

The CRAMP algorithm is an "exact" MPM solution for cracks, is efficient, and has advantages over other meshless methods for cracks. Here "exact" means CRAMP passes the "crack patch test" illustrated in Fig 1. A crack patch test involves solving a problem using any explicit crack algorithm that can also be solved by standard algorithms when the crack is defined by symmetry conditions alone. For example, Fig. 1 shows results for mode I double cantilever beam specimens (DCB) solved either by CRAMP (top) or by standard MPM with fixed displacements along the uncracked midplane of the specimen (bottom). CRAMP is an "exact" MPM method because the calculations in the top half of the CRAMP solution are mathematically identical to those in the standard MPM analysis of the symmetric problem. FEA with cracks in a mesh similarly passes a crack patch test (for opening cracks), but other meshless methods do not. Other meshless methods implement cracks by modification of influence zones around particles based on approximate node visibility or stress diffraction rules for how stresses are felt across cracks (Belytschko [10], Organ [11], Batra [12]). Although these approaches can include cracks, the use of modified interpolations, which often depend on arbitrary diffraction parameters (Organ [11]), means the methods do not pass a crack patch test.

2.2 Crack-Tip Fracture Parameters

Predicting crack propagation usually involves calculation of fracture parameters such as energy release rate or stress intensity factor. We have recently shown that CRAMP works well for calculation of *J*-integral, stress intensity factors, and crack-tip opening displacements (Guo [9]). Figure 2 shows typical MPM results for a double-edge notched plate impacted between the two

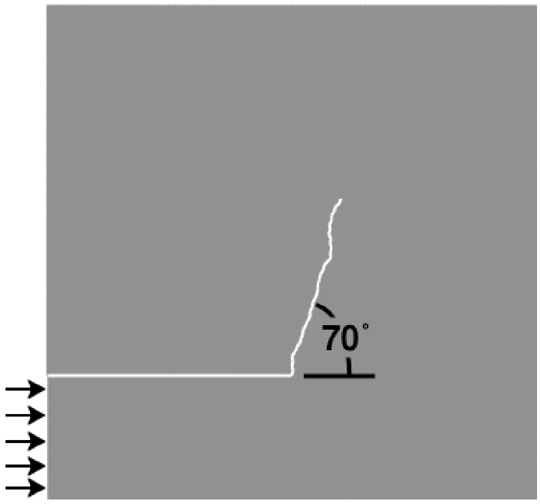

Figure 3: CRAMP prediction of crack propagation in the top-half of a DENP specimen. Crack propagation direction was predicted by maximum hoop stress criterion.

notches. The results show that MPM agrees well with prior numerical results on the same problem (Batra [12]). By choosing the *J*-integral contour to be along mesh lines of the background grid, the numerical integrations are efficient. Because CRAMP accurately tracks crack opening displacements it is possible to partition $J(t)$ into $K_I(t)$ and $K_{II}(t)$ stress intensity factors (Nishioka [13]).

2.3 Crack Propagation

The final step for full numerical modeling is to include crack propagation. Although MPM/CRAMP uses a background grid for efficient inclusion of cracks and calculation of *J* integral, the crack geometry is defined by massless particles that are not fixed to the grid. In other words, the crack description is meshless. This property makes inclusion of crack propagation trivial; it is only a matter of inserting a new crack particle. The crack can propagate in arbitrary directions unrestricted by the grid.

3 SAMPLE CRACK PROPAGATION CALCULATIONS

Figures 3, 4, and 5 show three crack-propagation examples using MPM/CRAMP. The materials were linear elastic and the analyses were isothermal. Failure was predicted by a maximum hoop stress criterion in which the crack propagates in the direction of the maximum hoop stress when that stress reaches a critical value. The crack-tip hoop stress can be calculated from $K_I(t)$ and $K_{II}(t)$; the critical hoop stress can be evaluated from an input K_{Ic} toughness (Belytschko [10]). Figure 3 shows crack propagation for the top-half of the specimen illustrated in 2. The mixed-mode loading caused crack growth at 70˚ from the crack plane. This prediction agrees with experimental results at low impact velocity (Kalthoff [14]). Figure 4 shows five disks under axial impact while constrained to remain aligned (Bardenhagen [4]). The mixed-mode loading for the angled crack under diametrical loading caused a curved crack path that ended at contact points between the disks. The modeling agrees with results of static experiments in diametrical compression (Shetty [15]). Figure 5 shows crack propagation in a two-phase composite in which

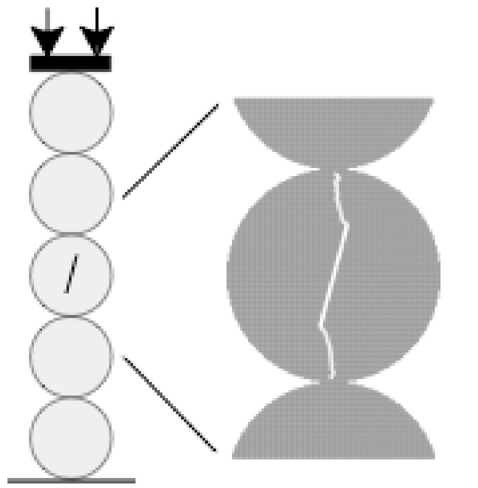

Figure 3: MPM calculation of crack propagation for five constrained disks under axial impact with a crack in the central disk.

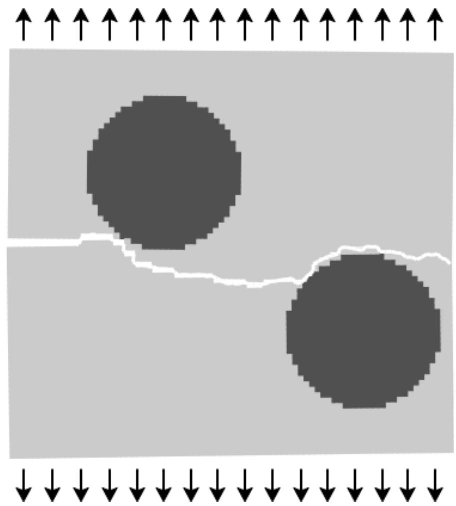

Figure 5: Crack growth though the compliant matrix of a composite with stiff inclusions.

the inclusions are much stiffer than the matrix. The crack path propagated through the matrix and around the inclusions. This crack path was a natural consequence of the maximum hoop stress failure criterion in a composite because no constraints were used to prevent propagation into the second phase or to promote propagation along the interface.

4 CONCLUSIONS

MPM/CRAMP can handle explicit cracks with the algorithmic efficiency of finite element analysis (FEA). FEA, however, has difficulty dealing with crack contact and severe problems dealing with crack propagation in arbitrary directions. MPM/CRAMP solves the crack contact problem by making use of prior contact methods developed for conventional MPM (Bardenhagen [4]). For crack propagation, MPM/CRAMP can exploit the particle basis (or meshless nature) of the method and easily grow cracks unrestricted by the grid. In summary, MPM/CRAMP combines the algorithmic efficiency of FEA for inclusion of explicit cracks with the advantages of meshless methods for handling crack propagation. It avoids the disadvantages of FEA for dealing with crack propagation and the weakness of other meshless methods for rigorously dealing with explicit cracks.

4.1 Acknowledgements

This work was supported by a grant from the Department of Energy DE-FG03-02ER45914 and by the University of Utah Center for the Simulation of Accidental Fires and Explosions (C-SAFE), funded by the Department of Energy, Lawrence Livermore National Laboratory, under Subcontract B341493.

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