Problem Statement

The “kinematic anomaly” is a term coined by Rebecca Brannon's group that refers to a pathological behavior in MPM in which simulations that are proceeding in an apparently stable fashion experience a “kick” in which a small region of the computational domain will experience non-physical acceleration, the source of which is under investigation. The simulation may proceed, or may go completely unstable. This behavior has been observed in the Uintah-MPM code, as well as an independently written code by another experienced practitioner. Here a vehicle for investigating this phenomenon is described in the hopes that other MPM enthusiasts will attempt to solve the same or similar and report their findings.

The 2D plane-strain problem described here is a variant of the so-called “breaking dam” problem, in which an initially stationary column of fluid is subject to gravity at T=0 and allowed to slosh down into the initially empty portion of the two-dimensional computational domain. All particles start with zero stress (i.e., the gravitational load jumps from zero to 9.81 m/s^2 at time zero). This problem seems to be particularly useful in exciting the pathological response, presumably because the fluid has no shear strength with which to resist aberrant acceleration; this observation might account for why similar anomalies are observed in materials having initial shear strength after the constitutive model imposes damage or melting to effectively reduce the material to an inviscid fluid. Figure 1 provides dimensions of the fluid column and computational domain. Grid cell size is 1 cm X 1cm, and 4 particles per cell (2x2) are used to describe the initial geometry. All domain boundaries are treated as planes of symmetry.

The pressure in the water is given by an equation of state described in Equation 64 in: "*On the Galerkin formulation of the SPH method*", L. Cueto-Felgueroso, et al. IJNME, 60:1475-1512, 2004. There, the (gauge) pressure P is given by, $P=K*(J^{-\gamma} - 1)$, where K is the low-pressure bulk modulus of the fluid, and J is the determinant of the deformation gradient, while $\gamma=7$ was used throughout these simulations. Alternatively, the pressure can be described by the Tait equation, $P = C*K*[\exp((1-J)/C)-1]$, where C is the universal Tait constant C = 0.0894 (this equation is assuming temperature-independent low pressure bulk modulus and thermal expansion coefficient). For problems with temperature variations, J should be replaced by J/J_{res}, where J_{res} is the determinant of the free thermal expansion deformation gradient.

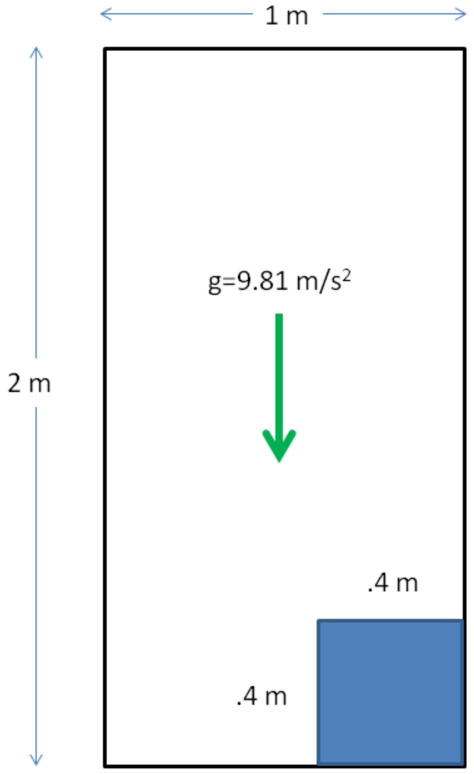

Figure 1: Schematic of the initial condition.

In the investigation described here, a range of bulk modulus values were used, 15 kPa <=K<=15 MPa. A Newtonian model for shear stress may also be included with viscosity set to 500 cP. Five seconds of physical time were simulated, although of the dozens of cases were considered, only a few of them actually survived intact for this duration. A CFL number of 0.2 was used to keep the time step within the expected stability constraints. Lowering this value makes little difference on the ultimate outcome. Dynamically adjusting the time step due to pressure-induced increases in wave speed has little effect for this problem, but it would be important for other problems that first compress the fluid before shearing.

A variety of pathological and confusing responses are observed in the results. Of course, the observations described here are visual in nature only, the metric of correctness is largely whether solutions "look right" and remain stable. One additional metric in measuring correctness comes from the pressure field, which should approach a hydrostatic distribution at late times. While an exact time evolving solution isn't known, several features that are known to be incorrect are described here:

1. With a small value of K, an initially beautiful simulation nearly always succumbs to a kinematic anomaly in which most of the particles are blown out of the closed domain in a matter of a few time steps.

2. While an increasing (and more realistic) value of K leads to simulations that survive for 5 seconds, and which look more or less realistic in terms of the final sloshing of fluid at that time, the pressure field in the fluid is extremely noisy with values that are many orders of magnitude outside of the expected hydrostatic distribution.

3. With small values of K, the very early (0.01-0.1 seconds) pressure distribution on the particles looks plausible, with a pressure gradient propagating from the lower boundary upward. However, with more physically realistic values of K, the pressure field immediately becomes "checker-boarded", in other words, noisy in a patterned way.

4. A decreasing (and more realistic) viscosity leads to less stable simulations.

Graphics demonstrating these behaviors can be supplied for comparison upon request.