

**FE 308**  
**Lecture 2 – Measurements and Errors**

**Types of measurements**

- 1.
- 2.
- 3.
- 4.
- 5.

**Survey Units of Measure**

- 1.
- 2.

**USCS – Horizontal and Vertical Distance**

**Foot** – The basic unit of measure

1893 – Adoption of the *survey foot*

$$39.37 \text{ inches} = 1 \text{ meter}$$

$$1 \text{ foot} = 0.3048006 \text{ meter}$$

1959 – Adoption of the *international foot*

$$1 \text{ inch} = 2.54 \text{ centimeters}$$

$$1 \text{ foot} = 0.3048 \text{ meter}$$

Difference is about 1 foot in 100 miles

U.S. public lands were surveyed prior to 1959. Thus, the old standard still applies.

Other common survey units:

**Rod** - 16 ½ feet (Also called a pole or perch).

**Chain** – 66 feet . 100 links in a chain. 4 rods in a chain. Also called a Gunter’s chain.

**Mile** – 80 chains.

**Vara** – about 33 inches. Old Spanish unit.

### English System – Horizontal and Vertical Angles

**Degree** – basic unit of measure for angles. 360 degrees in a circle

**Minute** – 60 minutes in a degree.

**Second** – 60 seconds in a minute.

Others:

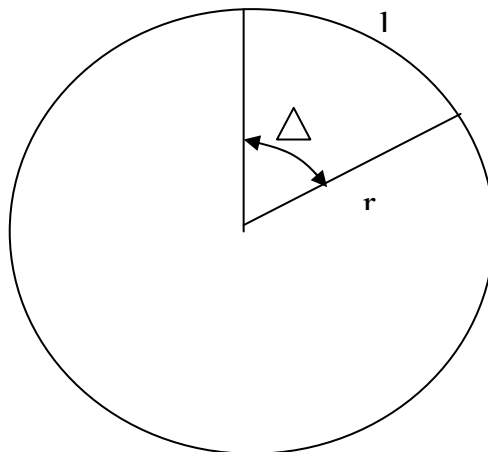
**Grads** – 400 grads to a circle (also called gons)

**Radian** – The angle subtended by an arc of a circle having a length = radius.

$$2 \pi r = \text{circumference}$$

$$2 \pi \text{ rad} = 360^\circ$$

$$1 \text{ rad} = 57^\circ 17' 44.8''$$



## Significant Figures

Definition:

Requires:

### Example

A steel tape with graduations to the 0.01' is used to make four repeat measurements of a distance. The recorded values are as follows:

100.04'      100.07'      100.06'      100.10'

The average of these record measurements is the sum divided by four.

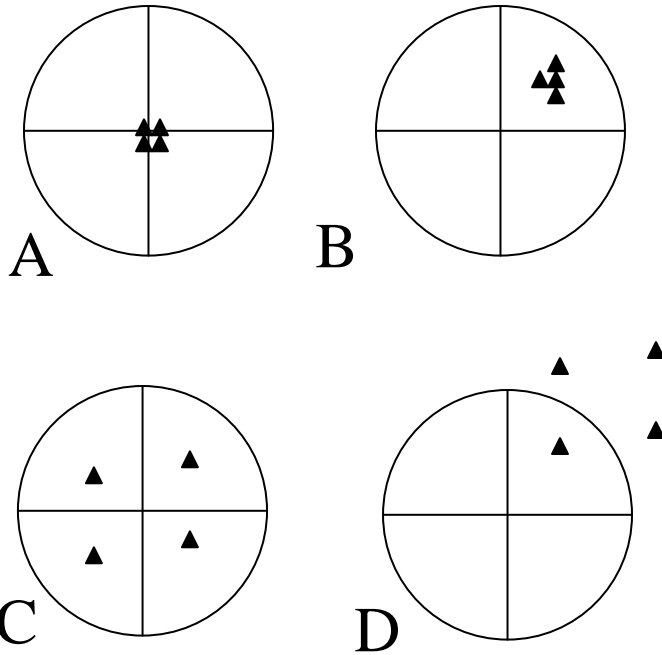
$$= 400.27 / 4$$

On your calculator, this can be expressed as 100.0675. However, this falsely implies that your measurement was precise to 0.0001'. The correct answer is expressed as 100.07.

The final digit is the rounded off or estimated value. This number is said to have 5 significant figures.

## Accuracy and Precision

How are these terms defined?



A.

B.

C.

D.

## Errors in measurements

### Measurement Error:

### Three types of measurement error:

1.

A.

B.

C.

Examples

A.

B.

C.

2.

A.

B.

C.

D.

Examples

A.

B.

3.

A.

B.

Examples

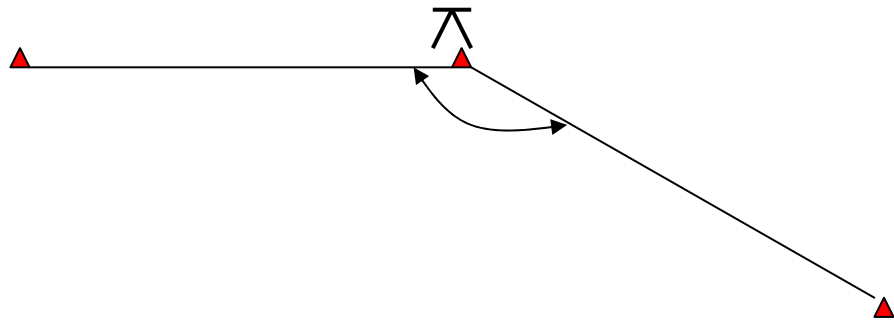
A.

B.

C.

### Measurement Probability

Measurements will fall into a *normal distribution* of probability around the true value.



If we measure this angle, there is a probability distribution that we will be somewhere near the *correct or true* value. This can be calculated as follows:

$$se = \frac{sd}{\sqrt{n}}$$

$$sd = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$$

The *standard error*, or *se*, is a measure of the variance of the *measured values* to the *true value*.

Interpretation:

The *standard deviation*, or *sd*, is a measure of the variance of the measurement values themselves.

Interpretation:

Both of these follow a normal distribution and can be described as follows:

For the standard deviation:

- 1.
- 2.

For the standard error:

- 1.
- 2.

As an example:

We measured a distance 6 times with a laser range finder and returned the following measurements:

37.29'  
37.25'  
37.37'  
37.18'  
37.32'  
37.45'

The average or *mean* distance is 37.31'

The *standard deviation* of the measurements is  $\pm 0.09'$ . In other words, we would expect that approximately 68% of the measurements would fall within  $\pm 0.09'$  of the mean distance or between  $37.22'$  and  $37.40'$ .

The standard error of the measurements is  $0.04'$ . In other words, we are 68% confident that the true average distance is within  $\pm 0.04'$  of the mean distance or between  $37.27'$  and  $37.35'$ .

### **Propagation of Errors**

Systematic errors propagate as the sum of the errors.

For example:

Random errors propagate according to Gaussian probabilities.

For example: