

A physically based, variable contributing area model of basin hydrology

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Abstract. A hydrological forecasting model is presented that attempts to combine the important distributed effects of channel network topology and dynamic contributing areas with the advantages of simple lumped parameter basin models. Quick response flow is predicted from a storage/contributing area relationship derived analytically from the topographic structure of a unit within a basin. Average soil water response is represented by a constant leakage infiltration store and an exponential subsurface water store. A simple non-linear routing procedure related to the link frequency distribution of the channel network completes the model and allows distinct basin sub-units, such as headwater and sideslope areas to be modelled separately. The model parameters are physically based in the sense that they may be determined directly by measurement and the model may be used at ungauged sites. Procedures for applying the model and tests with data from the Crimple Beck basin are described. Using only measured and estimated parameter values, without optimization, the model makes satisfactory predictions of basin response. The modular form of the model structure should allow application over a range of small and medium sized basins while retaining the possibility of including more complex model components when suitable data are available.

Un modèle à base physique de zone d'appel variable de l'hydrologie du bassin versant

Résumé. On présente un modèle pour la prévision hydrologique ce qui essaie de combiner les effets significatifs de la topologie du réseau des cours et les zones d'appel dynamiques aux avantages des modèles simples de bassin à paramètres composés. L'écoulement à réponse rapide est prévu d'un rapport stockage/zone d'appel qu'on trouve pour analyse de la structure topographique d'un groupe à travers un bassin. La réponse moyenne de l'eau du sol est représentée par un emmagasinement constant d'infiltration par écoulement et un emmagasinement exponentiel de l'eau souterraine. On complète le modèle en utilisant un processus simple du cheminement non linéaire ayant rapport à la répartition fréquence de connexion du réseau des cours ce qui permet de modéliser séparément des sous-groupes du bassin versant bien distincts, comme par exemple les zones des cours supérieurs et des pentes latérales. Les paramètres du modèle sont à base-physique au sens qu'on peut les déterminer directement en utilisant des mesures, et on peut se servir du modèle aux expositions pas jaugés. On décrit des processus pour l'application du modèle et pour des tests aux données du bassin versant de Crimple Beck. En utilisant des valeurs mesurées et estimées des paramètres seulement, sans optimisation, le modèle donne des estimations assez bonnes de la réponse du bassin. La forme modulaire de la structure du modèle devrait faciliter l'application tout à travers une gamme des bassins, petits ou de grandeur moyenne, tout en retenant la possibilité d'y incorporer des composants de modèle plus complexes quand on dispose des données convenables.

1. INTRODUCTION

There has been a recent change in direction in attempts to model the essential processes of delay and diffusion by which a basin modifies the pattern of rainfall contributing to discharge. From a position in which it was considered satisfactory to represent these processes by lumped analogues, it is now widely accepted that the dynamic spatial variation in discharge source areas must be taken into account more explicitly. As a result, it is also necessary that the effects of routing of discharge from the source area on the final distribution of basin outflow should be treated in more detail, even in small basins. In short, hydrological modelling is becoming more physically based. This paper is founded essentially on the view that there is a need for a simple physically-based model for medium sized basins, and, in particular, for a model with parameters that are directly measurable for a given basin. It should then be possible to avoid the derivation of parameter values for ungauged basins from regional statistical generalizations, e.g. Nash (1960), James (1972), Natural Environment Research Council (1975).

There is an immediate dilemma facing attempts to formulate such a physically based model. Every basin is an exceedingly complex open system with component processes and state variables that may change rapidly over space and time. Even if the processes operating were fully understood then an impossibly large number of parameters would be necessary to model the response of the spatially structured system in any but the crudest detail (Stephenson & Freeze, 1974; Beven, 1975). On the other hand, exceedingly simple models with only one or two parameters can provide a good empirical fit to the response of a particular basin (Nash, 1957; Dooge, 1959; Lambert, 1969). This paper presents a model for humid temperate areas that attempts to combine the advantages of simple lumped parameter models with the important distributed effects of variable contributing areas and flow routing through the channel network, while retaining the possibility of deriving parameters by direct measurement within the basin under study.

2. VARIABLE CONTRIBUTING AREA CONCEPTS

Runoff may occur in a uniform basin in at least four major ways.

(a) Rainfall intensity exceeds infiltration or storage capacity resulting in overland flow all over the basin. This is the classical version of Horton's (1933) model and is thought to have considerable relevance in areas of low vegetation cover and high rainfall intensity. However, in humid temperate areas with a vegetation cover, the measured infiltration capacities of soils are generally high in comparison with normal rainfall intensities (Kirkby, 1969; Freeze, 1972). In this case the Horton model of basin response is not applicable.

(b) Rainfall intensity exceeds infiltration or storage capacity on a variable area of near-saturated soils. This is the basis for Betson's (1964) partial area conceptual model in which it is recognized that the spatially variable nature of infiltration capacities and differences in moisture status at the soil surface, caused by downslope flow of water, will result in some parts of the basin being far more likely to produce infiltration excess overland flow than others. Engman & Rogowski (1974) have produced a relatively simple physically based model founded on this concept.

(c) Rain falling on stream channels and completely saturated soils. Where the latter are adjacent to stream channels (as is common) this source of overland flow contributes directly to the storm hydrograph (Dunne & Black, 1970). The zone of soil saturation may extend completely from bedrock or may build up above a relatively impermeable layer within the soil.

(d) Downslope lateral flow of saturated or unsaturated soil water. Most of this flow will be within the soil ('subsurface storm flow'), but it may locally exceed the soil storage capacity and return to flow over the surface at much higher velocities ('return flow', Musgrave & Holtan, 1964; Dunne & Black, 1970). Subsurface flow velocities are commonly too slow to contribute appreciably to the peak of the storm hydrographs although in volume terms subsurface flow may dominate the overall response of the basin in providing the hydrograph tail and low flows (Knisel, 1973).

In small humid temperate basins mechanisms (b) and (c) appear to be the critical sources of storm flow, with subsurface flow making a highly significant contribution in setting up the soil water conditions prior to further storm rainfall. These processes and their characteristics are thought to explain the observed nonlinearity of runoff in response to rainfall, and any simple physically based hydrograph model must reflect this general conceptual knowledge of the mechanisms involved.

A choice is available between an infiltration rate approach to the prediction of overland flow, as in the model of Engman & Rogowski (1974), and a soil storage based approach in which the infiltration rate is essentially considered to be non-limiting such that the prediction of overland flow occurs when storage capacity is exceeded. The latter approach has been adopted here both because it would appear to be more physically realistic in British basins and because it has operational advantages with respect to moisture accounting.

3. MODELLING CONCEPTS

A number of physically based deterministic models of the variable contributing area concept of basin response are reported in the literature (Knapp, 1974; Calver *et al.*, 1972; Freeze, 1972; Hewlett & Troendle, 1975). These models, of varying degrees of sophistication and methodological rigour, have been essentially based on distributed moisture accounting for soil elements within segments of hillslope. The data and computing requirements of these models are, however, so great as to restrict their practical application to research projects (Stephenson & Freeze, 1974) where economic criteria are less dominant. In formulating a simpler model, the present study attempts to integrate the important distributed effects described above with the simple lumped model of the average response of soil water storage in the basin. Simplicity is not held to be a virtue in itself but is a pragmatic response to a desire to produce a model that is capable of being applied operationally, whilst reflecting the current state of knowledge of hydrological processes.

Kirkby (1976a) provides an analysis of simple lumped storage models which form a basis for the present study. The effect of combining several linear stores of differing time constants in a series chain is particularly noted. It is apparent that the stores with the highest time constants have the greatest effect on the response of the system so that the simplest two-store approximation will mostly reflect the properties of the two slowest responding stores in a longer chain. If the stores are nonlinear, with time

parameters taken to represent average residence time in the storage element, this principle remains valid, as has been demonstrated by Wooding (1965). Furthermore, the slowest responding store must be most accurately modelled in terms of its non-linearity, because the outflow function is less sensitive to the form of the faster responding store provided an appropriate time parameter is used for it. A linear approximation to this store may commonly be sufficient.

Thus, for quick response flow, such as saturation excess overland flow produced on a variable contributing area, the duration of flow is likely to be critical only in small basins. Indeed, for larger ones where channel routing effects become increasingly important, overland flow may be treated as effectively reaching the channel within one time step. It is, however, very important to accurately model the quantity of quick response flow and the time at which it is produced. This will involve modelling the dynamic response of a variable characterizing the surface soil layer. The overall timing of infiltration will certainly be important, but the characteristics of the infiltration store will be less important than those of the subsurface store, which may directly shape the overall hydrograph. Nonlinear effects of the subsurface store, if apparent, should therefore be modelled carefully. Similar arguments apply to additional ground-water storage elements if these become necessary as larger basins are modelled.

The effect of the channel network probably becomes important for basins larger than about 10 km² where the time constant of the network (i.e. travel time through it) becomes as long as for the infiltration phase. However, arguments paralleling the above suggest that a linear network model may be sufficient in basins of less than 1000 km². In basins larger than 1000 km², routing time becomes as long as the subsurface response time. It is also suggested that the important effects of the channel link frequency distribution of a given network on the form of the outflow hydrograph should be taken into account (Surkan, 1974; Kirkby, 1976b). In fact it is convenient to use the channel network to subdivide the basin so that sub-basin areas of markedly different hydrological characteristics are modelled separately.

The exact structure of the model must necessarily reflect the types of hydrological characteristics that are quick, convenient and economic to measure for a particular basin. These include the topographic structure together with infiltration rates, overland and channel flow velocities, a small number of discharge measurements and some simple measurements of the soil hydrological characteristics. There will be a number of ways of interpreting these rather crude measurements in terms of the simple storage elements discussed above. As a first possibility a sub-basin model has been formulated from the following components (Fig. 1).

(a) A variable contributing area component related (by the nature of the processes outlined in the previous section) to subsurface soil water storage. Rain falling on the contributing area, A_c , will immediately become overland flow.

(b) A surface interception and depression store, S_1 , with a maximum value S_D , which must be filled before infiltration from it can take place. Evaporation is allowed from this store at the estimated potential rate until it is empty.

(c) A near surface infiltration store S_2 . A storage-based approach to infiltration has been adopted, with a constant leakage rate i_0 allowed from this store to the exponential subsurface store S_3 , in the area that is not considered saturated. Input to the store S_2 takes place (once the interception store S_1 is filled) at the rainfall rate i unless:

$$i > i_{\max} = i_0 + b/S_2 \quad (1)$$

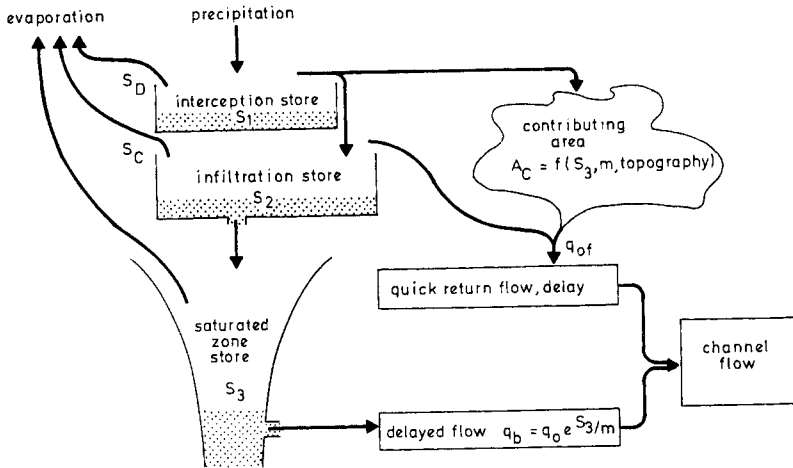


FIG. 1. A schematic representation of the sub-basin model.

In this case excess rainfall ($i - i_{\max}$) is considered to reach the basin outlet by a surface route (infiltration excess overland flow). If under extreme conditions a maximum value of near surface storage, S_c , is exceeded, then again excess water is considered to reach the sub-basin outlet by a surface route (saturation excess overland flow). Further comments on the infiltration component are included in section 8. Further losses due to evaporation are allowed from the store at a decreasing rate depending on the level of S_2 . Thus

$$e_a = e_r S_2 / S_c \quad (2)$$

where e_r is the potential evapotranspiration remaining once the interception store S_1 is depleted, and e_a is the actual loss from the infiltration store.

(d) A nonlinear subsurface saturated soil water store, which provides the delayed flow of Fig. 1. The simplest form of nonlinear store is an exponential store for which

$$q_b = q_0 \exp(S_3/m) \quad (3)$$

where q_b is the flow reaching the channel from the store, q_0 is the flow when $S_3 = 0$ and m is a constant. In the present model formulation this relationship is used such that S_3 is zero when the average soil water store (over the sub-basin) is just saturated. Positive values of S_3 therefore represent a moisture surplus and negative values a deficit (below average profile saturation).

This sequence of storage elements is assumed to represent the average response of the soil water in a homogeneous sub-basin unit. In this respect therefore, each sub-basin is treated as a lumped system. It is further assumed that the dominant source of quick return or surface flow is an area of surface saturation, or variable contributing area, the extent of which varies with the average level of subsurface soil water storage as represented by the store S_3 .

4. MODELLING A VARIABLE CONTRIBUTING AREA

In attempting to model the response of a basin in simple terms, the spatial and temporal variations of moisture must be grossly simplified. For a given average level of moisture

storage there will be a range of possible spatial distributions, even assuming spatially uniform rainfall. Thus, to integrate the distributed effects of a variable contributing area with a lumped model of average soil water response described above, some assumptions must be made about the duration of rainfall inputs. The simplest is to assume a time-independent steady state at a rainfall rate \bar{i} .

For a point, i , in the basin at which the area drained per unit contour length is a , the local slope angle is β and with a soil profile through which discharge per unit width at a point may be approximated by an exponential function of the form

$$q = K_0 \exp(S_i/m) \tan \beta \quad (4)$$

where q is the flow downslope under an assumed hydraulic gradient due to gravity alone and $K_0 \tan \beta$ is the flow at point i when the (relative) storage, S_i (in rainfall equivalent units, eg mm), is zero.

Then under steady state conditions

$$q = \bar{i} a = K_0 \exp(S_i/m) \tan \beta$$

or

$$S_i = m \ln(\bar{i} a / K_0 \tan \beta) \quad (5)$$

The saturated area may then be defined as the area for which $S_i > S_T$ or

$$a/\tan \beta > K_0/\bar{i} \exp(S_T/m) \quad (6)$$

for some local maximum storage value S_T . Over the whole basin of area A , for constant K_0 and m , mean subsurface storage is given by

$$\begin{aligned} S_3 &= 1/A \int_0^A S_i \, dA' \\ &= 1/A \int_0^A m \ln(\bar{i} a / K_0 \tan \beta) \, dA' \\ &= 1/A \int_0^A m \ln(\bar{i} / K_0) \, dA' + m \lambda \end{aligned} \quad (7)$$

where

$$\lambda = 1/A \int_0^A \ln(a/\tan \beta) \, dA'$$

is a constant of the basin, independent of K_0 and m . The value of λ may be approximated by $\lambda = \frac{1}{A} \sum \ln(a/\tan \beta) \Delta A$ where ΔA is some elemental area of constant $a/\tan \beta$ in the sub-basin.

Combining Equations 6 and 7, the saturated area is that for which

$$\ln(a/\tan \beta) > S_T/m - S_3/m + \lambda \quad (8)$$

The spatial distribution of $\ln(a/\tan \beta)$ can be readily mapped for a particular unit. Then if S_T can be assumed spatially constant, a single parameter $\lambda + S_T/m$ relates the topographic structure of the basin and average soil moisture storage to the saturated area. In a subsequent rainfall the area obtained in this may be closely identified with the concept of contributing area so that overland flow may be estimated as

$$q_{of} = i A_c \quad (9)$$

where i is an instantaneous rainfall intensity and A_c is the saturated area calculated from Equation 8. Obviously this procedure does not allow for the mechanism of

return flow described in section 2 but does introduce one major source of nonlinearity into the model. For larger basins taken as a whole, it may be expected that the slope of the contributing area/average storage function will be low at low values of S_3 , so that the condition for linearity (constant A_0) is more closely approximated. An example of the relationship for a small headwater area of the Crimble Beck, near Harrogate, North Yorkshire is given in Fig. 4c.

Early modelling studies at Crimble Beck suggested that even for small sub-basin areas, overland flow travel times were causing a significant delay in the timing of sub-basin discharge, a view that was reinforced by tracing experiments in the field. Consequently a simple overland flow routing routine has been included in the model based on the expected spread of contributing area in relation to the topography and a constant overland flow velocity parameter OFV. Then the time taken to reach the sub-basin outlet from any point within the predicted contributing area is given by

$$\sum_{i=1}^N \frac{x_i}{\text{OFV} \tan \beta_i} \quad (10)$$

where x_i is the length of the i th flow path segment of slope $\tan \beta_i$, and N is the number of segments between the point and the outflow. Given a value of OFV a unique time delay histogram can be derived from the basin topography for any value of the contributing area A_0 .

This concept of overland flow routing is somewhat similar to the time delay histogram method of unit hydrograph derivation of Clark (1945) but the present model allows that this histogram and consequently the time distribution of surface flow reaching the sub-basin outlet, may change dynamically with changing contributing area.

The problem of equifinality in the choice of model components, even where these are ostensibly physically based, lies heavily on models for hydrological forecasting. It is hoped that the theory underlying the present model combines components in a sensible, rigorous and realistic way. It is not, however, suggested that the model is capable of universal application, and indeed use of the present formulation may be restricted to medium scale (say $< 500 \text{ km}^2$) basins underlain by relatively impermeable bedrock.

5. CHANNEL NETWORK ROUTING

It was suggested above that the use of a complex channel routing algorithm would not be justified in the small to medium sized basins for which the present model is proposed. However, a study carried out on the channels of the Crimble Beck basin showed that the channel flow processes in the small, steep, rough and very irregular streams are highly nonlinear (Beven, 1976). Consequently a simple nonlinear routing algorithm was developed to complete the model, based on the velocity relationship.

$$c(t) = \text{CHA} Q(t)^{\text{CHB}} \quad (11)$$

where $Q(t)$ is the discharge at the outflow of the whole basin at time t , $c(t)$ is an average kinematic wave velocity for the network which is assumed to be spatially constant and equal to water velocities measured by tracer experiments. This may be a reasonable approximation in small rough channels in which there are numerous natural flow controls. The coefficients CHA and CHB are constants of the channel.

Equation 11 provides the basis for a nonlinear convolution routing procedure, that takes into account the distribution of predicted inflows with distance along the channel network from the outflow, with the form

$$Q(t) = \int_{\tau = -\infty}^t [q_1(\tau, x) c(t - \tau) + q_h(\tau, x)] d\tau \quad (12)$$

where $q_1(\tau, x)$ is the total lateral inflow to the channel at time τ and at distance

$$x = \int_{\tau' = 0}^{(t - \tau)} c(\tau') d\tau'$$

from the basin outlet, and q_h is the total headwater inflow to the channel at time τ and distance x .

This procedure must be used explicitly forwards in time and is not exact for the passage of a flood wave. It does specifically make use of the distributed nature of the network but does not allow for the diffusion of contributions to channel flow during passage through the channel system. It is thought, however, that the procedure may be sufficiently accurate for medium scale basins in which the response time of the channel network is less than that of the slope segments. When CHB is zero this channel routing procedure reverts to a linear channel representation of the network (Dooge, 1959; Kirkby, 1976b). Further work on channel routing in small, steep, rough channels has shown that this procedure may not always prove satisfactory, and either a linear channel representation, or a more complex kinematic method (e.g. Li *et al.*, 1975), should then be used.

6. THE APPLICATION OF THE MODEL TO THE CRIMPLE BECK BASIN

The headwaters of the Crimple Beck, a tributary of the River Nidd, lie to the south-west of Harrogate, Yorkshire (Fig. 2). The Crimple Beck basin covers some 8 km², its lower limit being a flat-vee Crump weir installed by the Yorkshire Water Authority (grid reference SE 284518). The basin ranges from 115 to 250 m OD and has an average annual precipitation of about 800 mm. The underlying bedrock is predominantly sandstones and intercalated shales, mostly overlain by Pleistocene glacial tills (0.25–1.5 m thick) with periglacial and fluvial deposits in the lower part. The drainage network is incised into these deposits and bedrock sections are rare. At least two local bedrock aquifers in the area give rise to minor springs. The quality of the superficial deposits varies greatly from unweathered impermeable blue glacial clay to local lenses of almost pure sand found near the upper watershed. Over most of the area, however, the subsoil has been shown to be relatively impermeable.

The soils have been mapped in detail only in the Lanshaw and Tateback sub-basins. The patterns found in these areas have been confirmed by sparser data from elsewhere and reflect the nature of these parent materials and the drainage conditions determined by the degree of slope at a point. Most of the relatively flat upper slopes are characterized by stagno-gley soils, especially where the bedrock is shale or till material. On steeper slopes and over sandstone the better drainage conditions result in soils that vary from humic brown podsols to typical brown earth soils. There is some evidence in the headwaters for soils with interbedded layers of peat and stony sand developed over head or clay till.

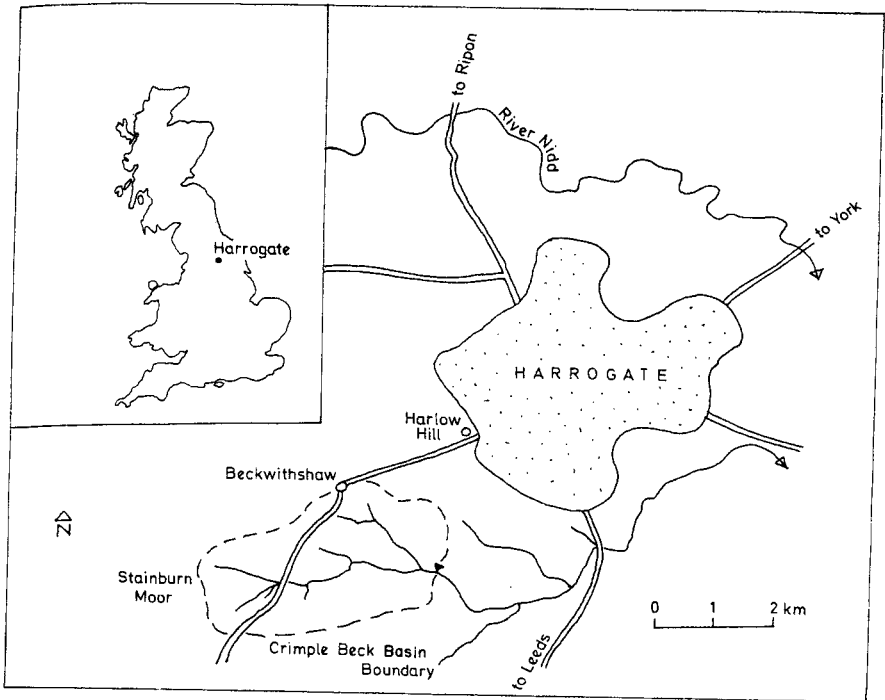


FIG. 2. Location map for the Crimple Beck study area.

The vegetation reflects the most intensive agricultural use to which most of the area has been subjected. Much of the basin is under improved pasture for sheep and cattle grazing, with a grass/summer barley rotation on some areas of better soil. Areas of other crops and woodland (mostly mixed deciduous trees along the channel banks) are small. Large areas of the lower part of the basin are underdrained, mostly by mole and tile drains but with some stone drains dating back to the nineteenth century (Harris, 1975).

In the highest part of the basin (Stainburn Moor) there is an area of unimproved rough grazing dominated by *Nardus* and *Molinia* grasses. There are *Sphagnum* and other bog mosses and *Juncus* rushes in poorly drained marsh and flush areas, and heather communities on the drier sand lenses noted above.

For the simulations described in this paper Crimple Beck was divided into 23 relatively homogeneous sub-basins on the basis of the channel network and patterns of soils and landuse (Fig. 3).

7. MODEL CALIBRATION: THE ANALYSIS OF SUB-BASIN TOPOGRAPHY

The first stage in model calibration is to derive the value of λ and the distribution of $\ln(a/\tan \beta)$ with area for each sub-basin. To achieve this the spatial distribution of slope values must be obtained and the pattern of subsurface flowlines in the horizontal plane must be inferred for the subcatchment from the evidence of maps, air photographs or field observation. Experience during this study and elsewhere (Dunne *et*

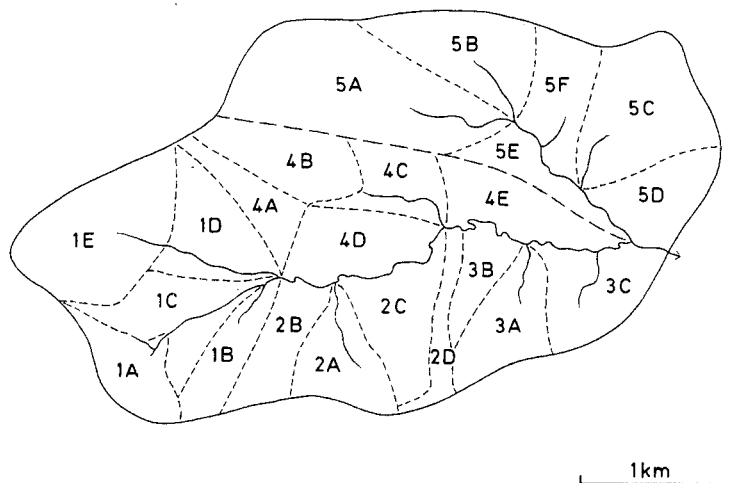


FIG. 3. The subdivision of the Crimple Beck basin into sub-basin units.

al., 1975) suggests that a period of field work, however brief, is invaluable in defining the convergent flowlines due to small variations in topography that may be important in producing the lowest values of contributing area. This work may be usefully combined with a programme for measuring slope gradients. Air photographs have also proved exceedingly useful in improving the data coverage over large areas. Maps rarely have the resolution to provide all the data required in more than a very approximate sense, but are always the starting point for this form of topographic analysis.

With these data in hand there are two approaches to the required analysis of topography. The first is to calculate values of a (the area drained per unit contour length at a point), and thence the value of $\ln(a/\tan \beta)$ for a large number of points in the sub-basin for which the slope angle is known. The definition of the area contributing to each point is, however, extremely tedious and a less demanding computerized method has been developed. In this approach the sub-basin is split up into a large number of elements by means of flowlines along lines of greatest slope and contour lines across the slope (Fig. 4). Each element is then defined by the coordinates of the three or four nodal points at its corners and a value of $\tan \beta$ is supplied to the program for the outflow edge of each element. Values of $\ln(a/\tan \beta)$ are computed by the program for the downslope edge of each element, together with the value of the parameter and the distribution of $\ln(a/\tan \beta)$, with area in the sub-basin. A map of the values of $\ln(a/\tan \beta)$ for the sub-basin of Fig. 4 is shown in Fig. 4b and compares favourably with observed patterns of saturated area (Fig. 5). Similar agreement has been found for other small instrumented basins (Kirkby *et al.*, 1976). The subdivision of the sub-basin should involve a sufficient number of elements for adequate definition of the flow pattern. The program also calculates a histogram of flow path lengths for the predicted saturated area at different values of $\ln(a/\tan \beta)$ for use in the overland flow routing procedure of the model.

This method of analysis is necessarily less accurate than analysis by hand due to the cruder analogue of topography represented by the elements on which the calculations are based. However careful choice of the element discretization should minimize the inaccuracies and if the time savings allow closer field investigation then a slight loss of accuracy might prove worthwhile.

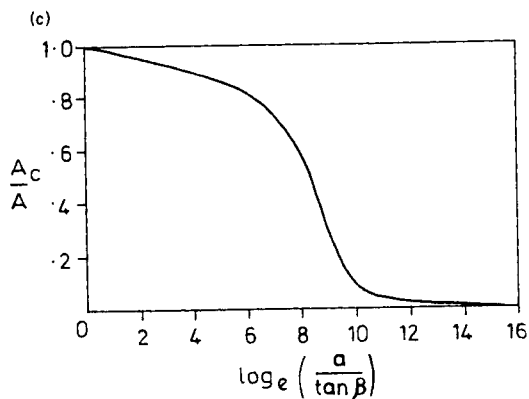
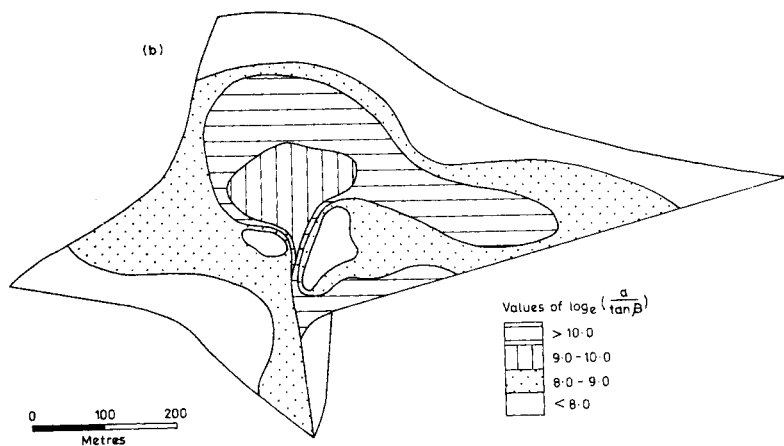
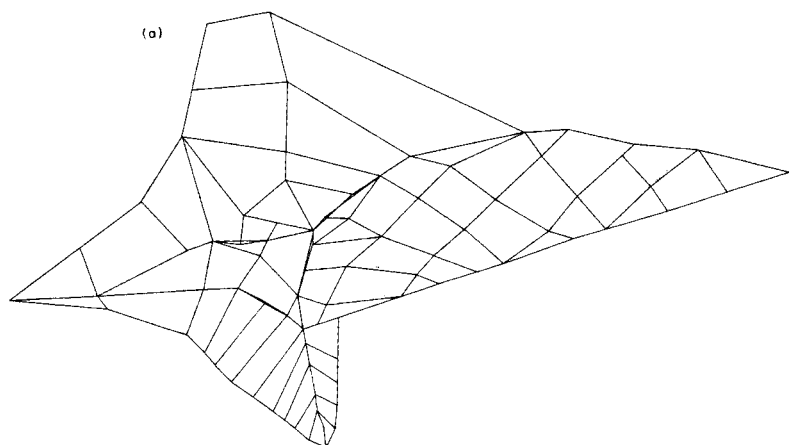


FIG. 4 (a) The discretization of the Lanshaw sub-basin (area 1A). (b) The spatial distribution of calculated values of $\ln(a/\tan \beta)$ in the Lanshaw sub-basin. (c) The cumulative distribution of contributing area A_c exceeding given value of $\ln(a/\tan \beta)$ for the Lanshaw sub-basin.

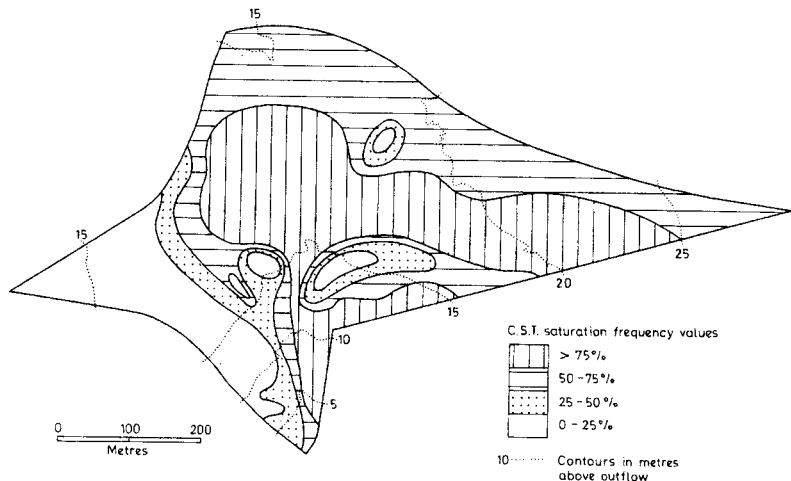


FIG. 5. Surface saturation map for the Lanshaw sub-basin (taken from Kirkby *et al.*, 1976).

8. MODEL CALIBRATION: OVERLAND FLOW, INTERCEPTION AND INFILTRATION PARAMETERS

The parameters S_D , S_c , i_0 , b and OFV may all be obtained by field experiment using a sprinkling infiltrometer. The apparatus used in this study satisfied the requirements of being relatively portable, with reasonably even ground cover over the plot size of 1×0.6 m and minimal wind effects. The rates of application that could be achieved (30–200 mm/h equivalent over area of 0.6 m^2) were, however, high in relation to the range of recorded rainfall rates.

The procedure adopted is as follows.

(a) One or more soil cores are taken adjacent to the chosen site, for analysis of near surface soil water storage (M_1) prior to the experiment.

(b) The spray is started and the time noted. The constant rate of application of water can be varied between tests by altering the height of the spray boom, the type of spray nozzle or the pressure at which water is supplied to the nozzle.

(c) Overland flow produced on the plot is collected and measured for given time periods so that the rate of runoff per unit area of plot can be calculated. The sprinkler is run until this rate approaches a constant value. An estimate of the velocity of surface runoff may be obtained by the addition of a small quantity of liquid tracer (such as fluorescein or salt solution) at the upslope end of the plot. The mean time of travel may be estimated by eye for visual dyes, or more accurately by the analysis of runoff samples taken at known times (or more conveniently for salt solution with a conductivity meter). A measurement of surface slope, $\tan \beta$, at the site allows the parameter OFV to be calculated from

$$\text{OFV} = x/(\bar{t} \tan \beta) \quad (13)$$

where \bar{t} is the average time of travel and x is the length of plot.

(d) The spray is stopped and once the runoff has ceased further soil cores are taken from the centre of the plot for analysis to determine moisture status (M_2) after the experiment.

(e) Once all the rates of flow and soil moisture values have been calculated, a graph of runoff rate and infiltration rate (as calculated from sprinkler application rate-runoff rate) is drawn as shown in Fig. 6. In the simplest form of analysis the parameter i_0 is taken as the final constant infiltration rate. The maximum value of the interception store, S_D , is taken as the amount of water applied before the constant runoff rate is achieved minus the change in near surface soil water storage ($M_2 - M_1$).

Estimation of the parameters b and S_c requires a more complex procedure involving several spray tests carried out at different application rates. Under a constant rate of input, i , to the near surface store S_2 (as in the sprinkler test) surface flow may occur either as a result of the infiltration capacity, i_{\max} , or the total storage capacity, S_c , being exceeded (see section 3). If T is the time from the start of the test to the onset of overland flow and T' is the time required to fill the depression storage S_D (that is $T' = S_D/i$) then for infiltration excess overland flow

$$i_{\max} \leq i$$

or from Equation 1

$$i_0 + b/S_2 \leq i \text{ when } S_2 = (T - T')(i - i_0)$$

This assumes that the depression storage S_D is satisfied first, or

$$i \geq i_0 + \frac{b}{(T - T')(i - i_0)}$$

or

$$T - T' = \frac{b}{(i - i_0)^2} \quad (14)$$

For saturation excess overland flow, the storage S_2 at the onset of overland flow is equal to S_c :

$$S_c = (T - T')(i - i_0)$$

or

$$T - T' = \frac{S_c}{(i - i_0)} \quad (15)$$

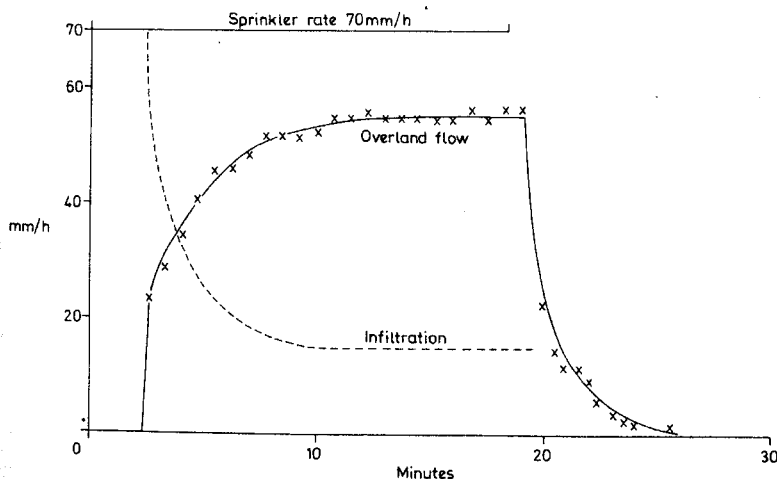


FIG. 6. Results from a sprinkling infiltrometer experiment, Lanshaw sub-basin.

Plotting $(i - i_0)$ against $\log(T - T')$ for tests at different application rates should distinguish the two mechanisms of overland flow generation, and enable the parameters b and S_c to be determined from the intercepts on the $(i - i_0)$ axis (Fig. 7). However, infiltration rates measured in the field proved to be generally very high in the Crimple Beck basin. In some spray tests no surface flow was recorded despite sprinkler application rates in excess of 150 mm/h. This has the result that when measured values of the parameter i_0 are used in the model simulations the infiltration store as a whole has little or no effect on the predicted discharges, and does not provide a delay before flow reaches the subsurface store. The parameters b and S_c are inoperative throughout the simulations. Thus, the infiltration store component of the model cannot be considered as satisfactorily tested. However, such infiltration rates do give some further support for the theory of subsurface saturation on which the variable contributing area component of the model is based. In order to partially avoid the effects of a lack of delay in the unsaturated zone it was felt that it was not realistic to use a time step of less than 3 h in the model calculations. It is recognized that with such a time step the influence of the fast acting channel routing component on the model results will not be tested satisfactorily. Measured rates of surface flow however were such that the influence of surface flow routing could extend over several time steps.

9. MODEL CALIBRATION: SUBSURFACE STORAGE PARAMETERS

The parameter m of the exponential subsurface store is estimated by making discharge measurements at the subsurface basin outlet during a rainless winter recession period when the effects of evapotranspiration are minimized. The most convenient way of obtaining such measurements is by the dilution gauging method, since the apparatus is readily portable and does not involve the installation of control structures. However, any existing discharge measurements within the basin can be incorporated in the analysis. The discharge measurements are converted to millimetres per unit area equivalent from which an estimate of change in average storage level in the sub-basin can be calculated during the period of measurement.

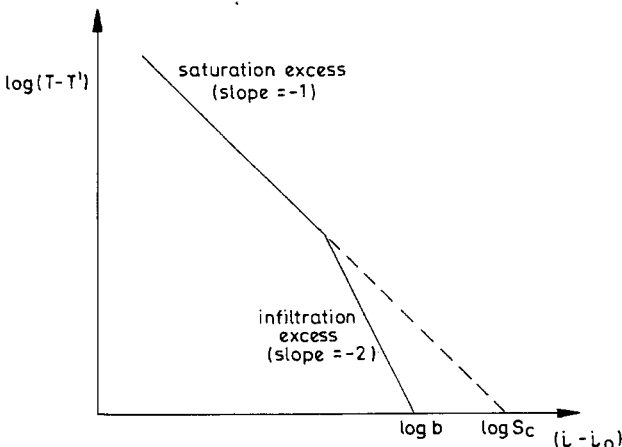


FIG. 7. Expected nature of results from infiltrometer experiments interpreted in terms of the storage based infiltration component of the model.

If the subsurface contribution to basin drainage conforms to the exponential function that is assumed in the model, then a graph of calculated relative storage level (as the abscissa) versus \ln (measured discharge) should plot as a straight line with a slope of $1/m$. Experience in this study suggests that the exponential store can provide a reasonable representation of the storage characteristics of sub-basins (see Fig. 8) with an improvement in fit with increasing area. Several sub-basins in the Crimble Beck area were shown to exhibit breaks in the slope of the storage versus \ln (discharge) plot at a fairly consistent discharge (a feature noted by Lambert, 1972, in the development of his ISO function model). Sub-basins also showed some scatter in the values of m . Both characteristics are shown in Fig. 9. Given a value of m for a sub-basin, the subsurface parameter q_0 can be calculated from measurements of average soil water storage relative to saturated storage obtained from soil cores taken throughout the soil profile at a number of locations, and at a known value of discharge per unit area from the basin. Substitution in Equation 3 remembering that S_3 represents average deficit below saturation then yields a value of q_0 .

The value of the final sub-basin parameter S_0 , the initial value of S_3 at the start of a simulation run, can be found by substituting a known or estimated value of actual discharge from the basin into Equation 3, given q_0 and m . This procedure is only satisfactory if the simulation run starts during a recession period such that all sub-basin

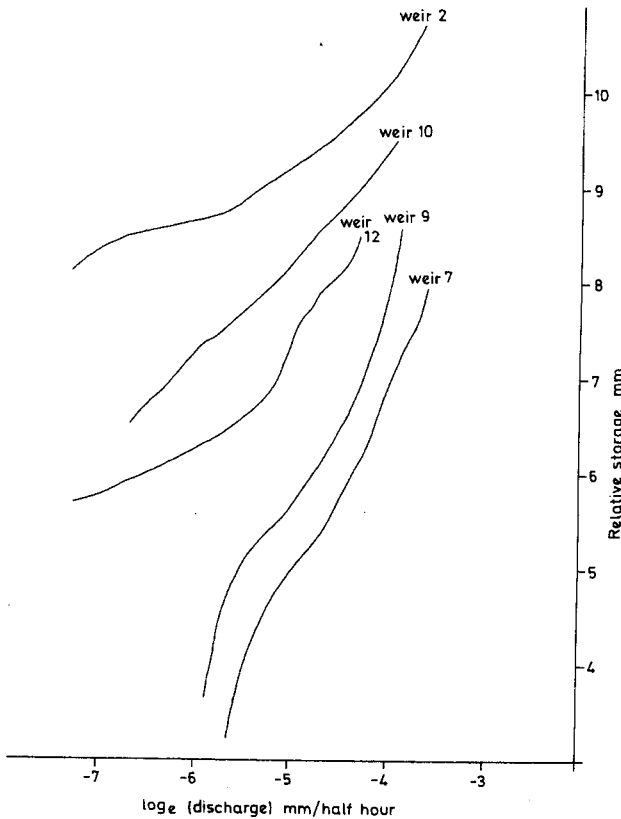


FIG. 8. Recession curves for different sites in the Crimble Beck basin as storage/ \ln (discharge) plots.

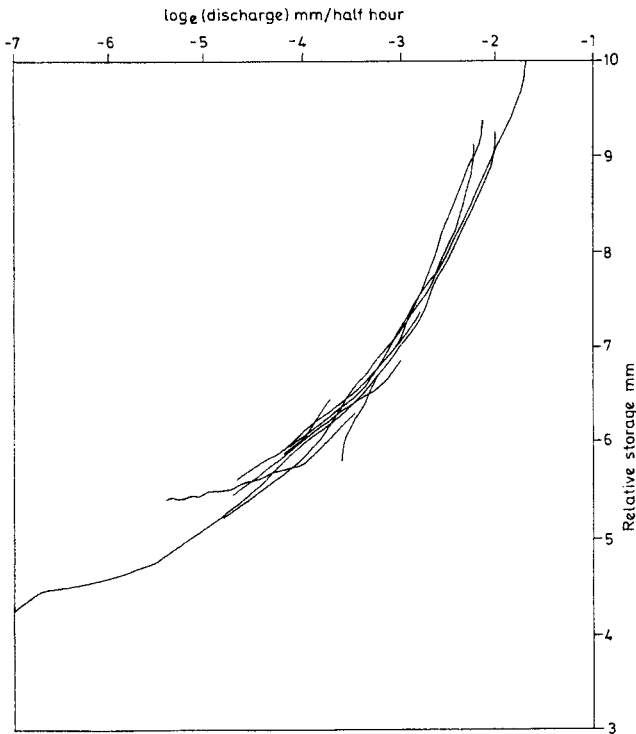


FIG. 9. A number of recession curves from the Lanshaw sub-basin as storage/ \ln (discharge) plots, the curves being matched by eye.

discharge is derived from subsurface drainage and both the interception and infiltration stores may be assumed to be dry. Errors in the initial conditions are quickly damped out and have been found to have little effect on the simulations.

10. MODEL CALIBRATION: CHANNEL ROUTING PARAMETERS

Very little information is available on channel flow parameters in small steep rough streams of the type characteristic of the basin under study. Deep slow pool sections alternate with numerous obstructions and disturbances to the flow. A study carried out in Crimple Beck showed that both spatial and temporal variations were complex and should not be described by the type of uniform flow relationship commonly used in larger streams (Beven, 1976). Channel travel times were very slow at low flows, with velocity increasing rapidly with discharge. It was felt that the relationship between velocity and discharge defined in Equation 11 would be a suitable approximation for the network routing procedure of this model. It will not generally be possible to obtain average measurements for the whole network. Flow velocity measurements were made at a reach close to the centre of the network for different flow stages.

Plotting these average velocity measurements against discharge from the whole basin (Fig. 10) showed a relationship of the form of Equation 11 for which the parameters CHA and CHB can be obtained.

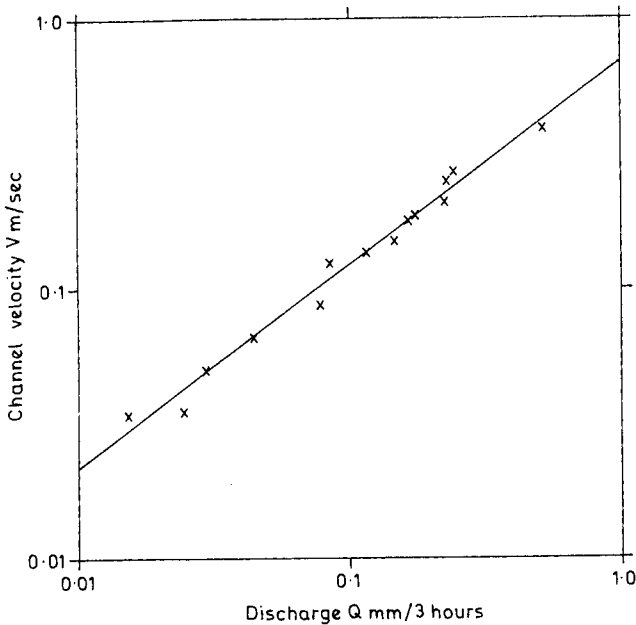


FIG. 10. Plot of average channel flow velocity measured by dilution gauging at a site in the centre of the basin, against discharge from Crimple Beck.

11. MODEL SIMULATIONS FOR CRIMPLE BECK SUB-BASINS

Initial testing of the sub-basin model was carried out on three areas of the Crimple Beck basin chosen to represent the range of conditions found in the area. Rainfall and discharge data were available for all three areas, and open pan evaporation estimates were available from the meteorological station at Harlow Hill just outside the basin (grid reference SE 290542, see Fig. 2). The Lanshaw sub-basin (area 1A of Fig. 3) is a convergent headwater area of rough grazing at the highest part of the basin (Fig. 4). The Tateback sub-basin is an area (0.25 km²) of improved pasture while the Hemingway sub-basin (area 4C of Fig. 3) is a sideslope area (0.233 km²) of improved pasture, also partly underdrained.

The parameters of the model were derived for each area by the procedures outlined in previous sections and the model run for both winter and summer periods. No optimization of the parameter values was carried out, the observed discharge records for each sub-basin being used purely for comparison with the discharge values simulated by the model. Some of the results from these simulations are shown in Figs 11, 12 and 13, and demonstrate that the model can provide a reasonable reproduction of the hydrological response of these small and very flashy areas. Comparison of Figs 12 and 13 shows that the simulations are much better for the winter period with a tendency to overpredict discharge later in the year. This suggests that the prediction of losses to evapotranspiration could be improved in the model.

As part of the initial testing of the sub-basin model, an analysis of the sensitivity of the simulations to variations in the parameter values (around the estimated values) was carried out. This confirmed that the infiltration parameters S_c , i_0 and b had no effect on the simulation for these particular basins within their measured range. The

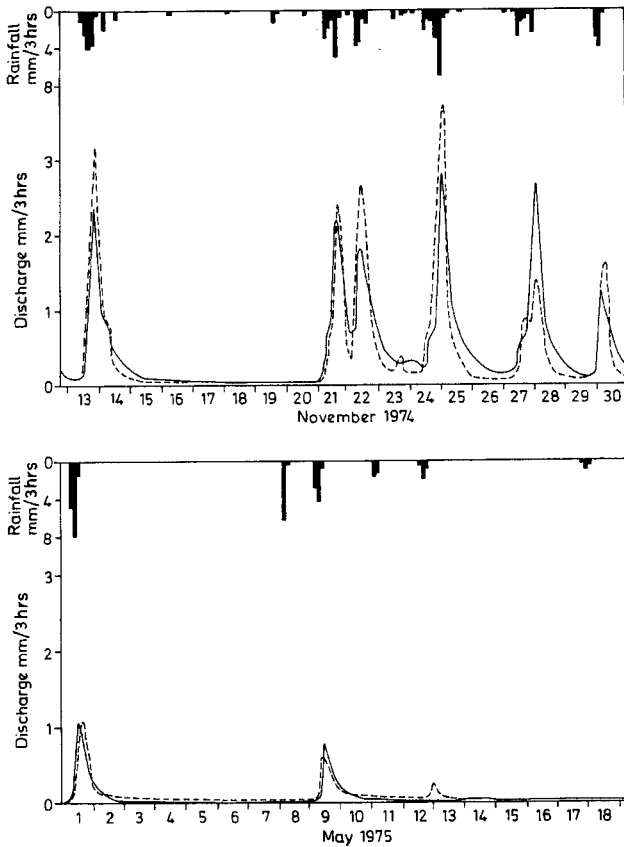


FIG. 11. Simulation results from the Lanshaw sub-basin part of winter and spring runs, 1975.

absence of a delay and additional topsoil losses to evaporation before infiltrated water reaches the saturated subsurface store, which the infiltration component of the model was intended to provide, must reduce the goodness of fit of the simulation, and, as noted above, precludes the use of the model with short time steps. There is no doubt that optimization of the parameters of a similar or alternative delay function could improve the model fit but would result in parameter values that would be difficult to interpret in physical terms. Fortunately, in the Crimple Beck basin, much of the storm flow at the main outflow weir appears to derive from quick response surface runoff, so that the lack of delay has had a relatively small effect on the simulations. However, as the subsurface flow contribution to discharge becomes relatively greater, the model would require improvement to cope with the increasing delays between rainfall and peak discharge.

As a result of the importance of surface flow in Crimple Beck, the value of the parameter m proved to be by far the most important. With both the $\ln(a/\tan \beta)$ distribution and thence the value of λ considered as constant quantities for the sub-basin, the parameter m , through the relationship of Equation 8, influences both the simulated area of surface saturation at a given level of storage, and the increase in storage required to achieve a given increase in contributing area. Effectively an increase in m will reduce the proportion of rainfall that reaches the channel by a surface route.

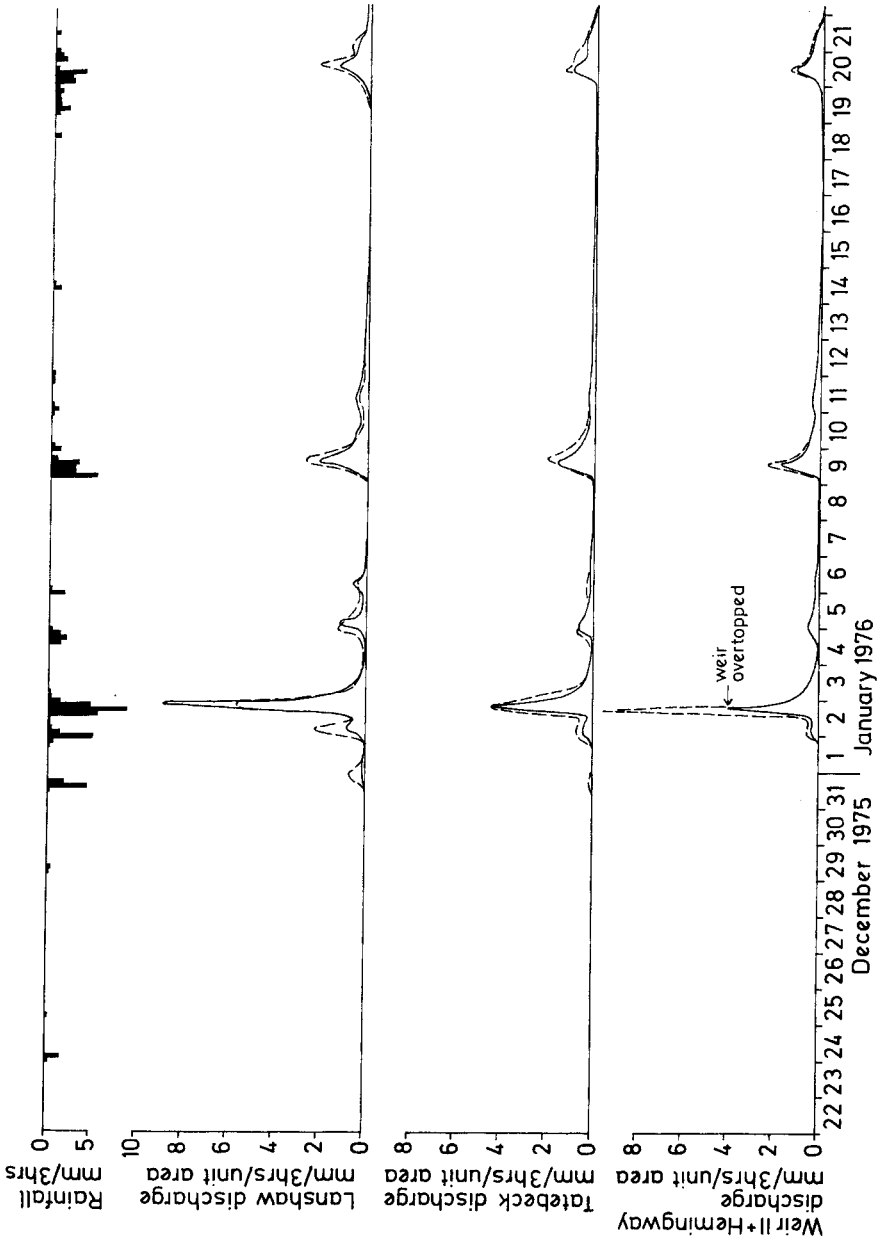


FIG. 12. Simulation results for the Lanshaw, Tateback and Hemingway sub-basins, part of winter run, 1975/76. Period 1: plot intervals 352-600.

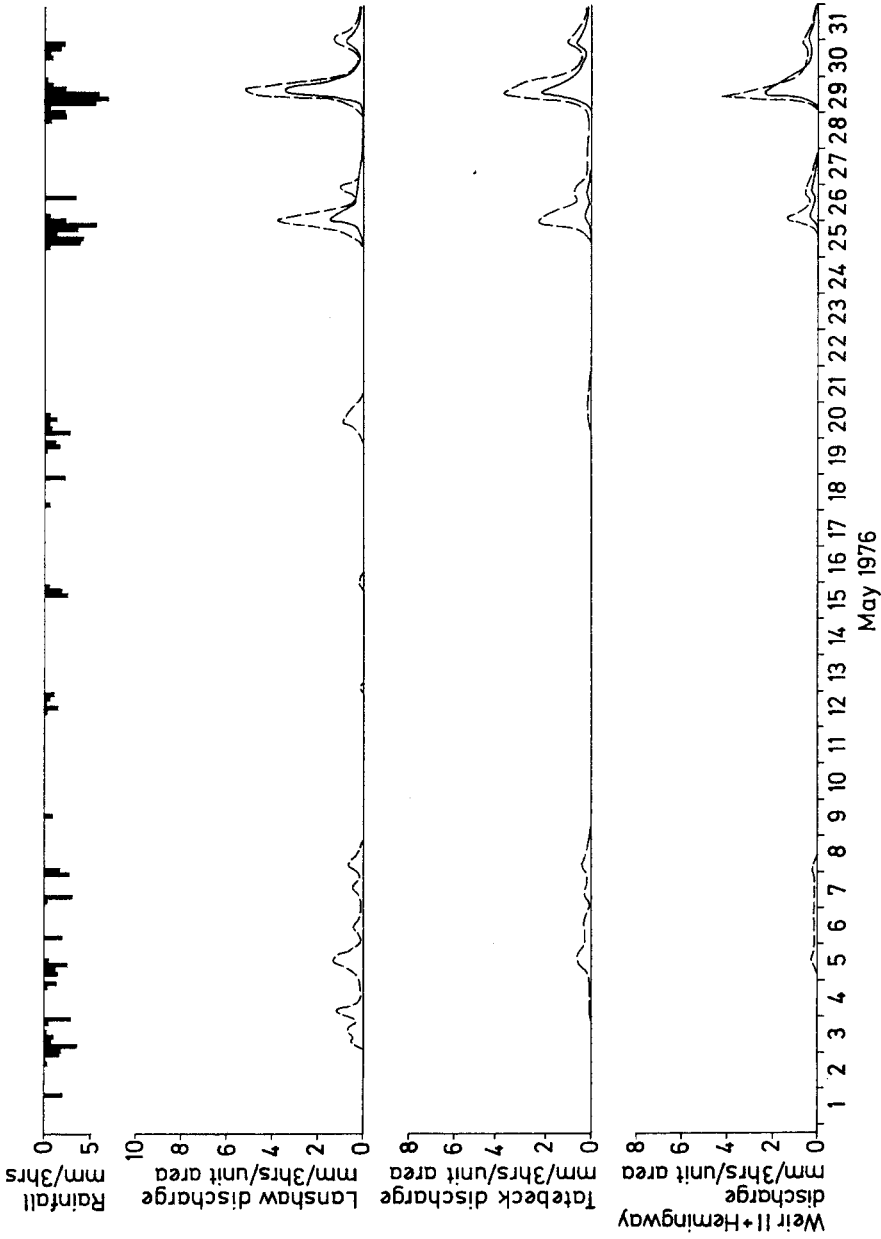


FIG. 13. Simulation results for the Lanshaw, Tateback and Hemingway sub-basins, part of spring run, 1976. Period 2: plot intervals 1398-1643.

However, the relationship between m and other parameters, as revealed in the goodness-of-fit response surface used in the assessment of parameter sensitivity, is complex. Three measures of goodness-of-fit were used as follows:

$$F_1 = \sum_{j=1}^N \sum_{i=1}^{M_j} (\text{QOBS}_j - \text{QSIM}_i)^2 \quad (16a)$$

$$F_2 = \sum_{j=1}^N \left(\sum_{i=1}^{M_j} \text{QOBS}_i - \sum_{i=1}^{M_j} \text{QSIM}_i \right)^2 \quad (16b)$$

$$F_3 = \sum_{j=1}^N (\text{QOBS}_{\text{pk}} - \text{QSIM}_{\text{pk}})^2 \quad (16c)$$

where QOBS_i and QSIM_i are the observed and simulated discharge values at time step i , N is the number of storm periods in the simulation, M_j is the number of time steps in the j th storm period and QOBS_{pk} and QSIM_{pk} are the observed and simulated peak discharges for each storm period. All three indices become smaller as the fit between observed and simulated record improves. Discussion of these and other goodness-of-fit indices are given by Plinston (1971), Aitkin (1973), and Douglas (1974), but note that the indices are not being used as objective functions for optimization of the model parameters, which are fixed at their measured values. No one index is entirely satisfactory. F_1 , an overall sum of squared errors index, has been the most commonly used in hydrological simulation studies, despite the fact that small errors in timing between observed and simulated hydrographs can result in relatively high values of F_1 . The other two indices avoid this, but F_2 , a measure of agreement between total storm discharges, is insensitive to hydrograph shape, and F_3 , while sensitive to the important fit between peak discharges, may not reveal other gross errors. However, all the indices, combined with careful visual inspection of the runs have value in indicating how far a simulation may be considered satisfactory. Certainly, an analysis of model sensitivity based on only one such index could be misleading.

It was found that the measured parameter values do not represent the optimum value. Indeed, optimization of the parameters m and q_0 would lead to values that eliminate the use of the contributing area concepts of the model and transform the model into a pure exponential storage representation of the basin. This is confirmed by a view of the F_1 response surface (Fig. 14) which shows the markedly reduced sensitivity to the value of m on one side of the optimum parameter values once the contributing area component of the model has negligible effect on the simulation.

Figure 14 also shows that the chosen value of q_0 is important. This parameter has the primary function of adjusting the outflow from the exponential store as calculated using average storage and the estimated value of m , to be equivalent to the subsurface outflow from the basin as a whole. However, since this will affect the level of average storage, it will also influence the contributing area component of the model.

The remaining subsurface store parameter is the initial condition S_0 . It has already been noted that this parameter has a relatively small effect on the simulations due to feedback mechanisms in the operation of the model, as is shown by the response surface of Fig. 15a where the effects of variations in the parameters S_0 and S_D are illustrated. Model simulations are also relatively insensitive to the parameter S_D for the winter period on which Fig. 15a is based. S_D governs the total losses before either infiltration or surface runoff will occur and it is an important control of predicted losses to evaporation from the model, where the potential losses between rainstorms

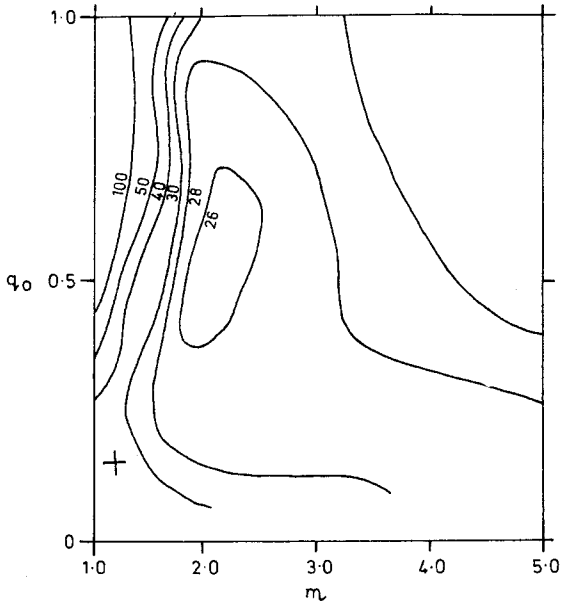


FIG. 14. F_1 response surface for the parameters m and q_0 . Lanshaw sub-basin November/December 1974 simulation. Cross represents estimated values.

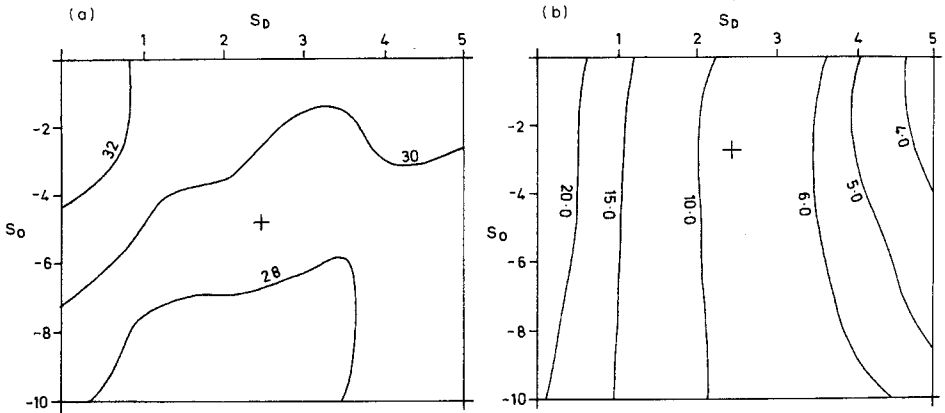


FIG. 15. (a) F_1 response surface for the parameters S_D and S_0 . Lanshaw sub-basin, November/December 1974 simulation. Cross represents estimated values. (b) F_1 response surface for the parameters S_D and S_0 . Lanshaw sub-basin, March/April 1975 simulation. Cross represents estimated values.

exceed the value of S_D . It would therefore be expected that the model would exhibit greater sensitivity to this parameter during summer months when potential evaporation rates are higher, and this is confirmed in Fig. 15b. This figure also suggests that an optimum value of the parameter might also exhibit seasonality. To an extent such seasonal effects must reflect the physical reality of basin hydrological response but could also be compensating for deficiencies in the model formulation with the suggestion that it is capable of further improvement.

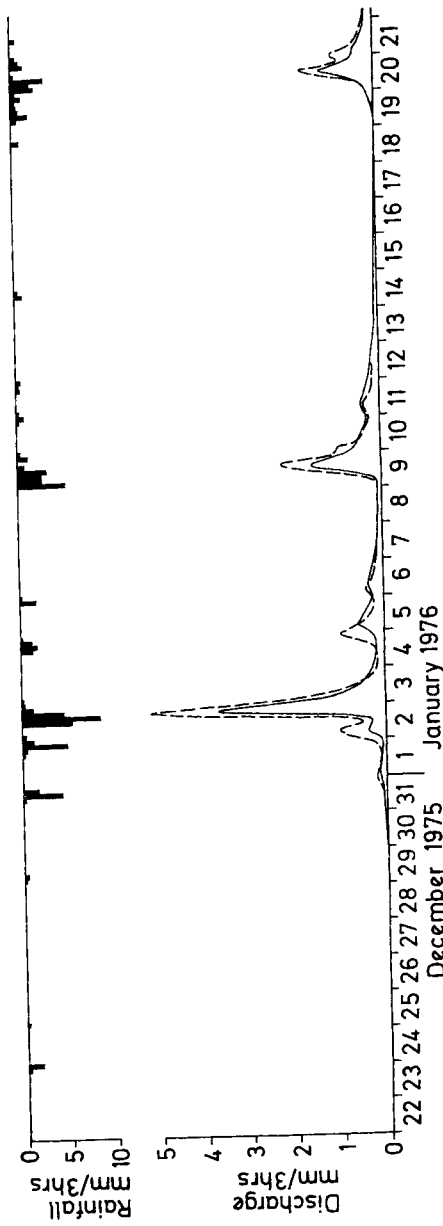


FIG. 16. Simulation of the Crimple Beck basin, using all sub-basin simulations and the channel network routing algorithm. Part of winter period, 1975/76.

TABLE 1. Measured and estimated model parameter values for headwater and sideslope sub-basin areas of Crimple Beck

| Area | S_D (mm) | i_o^* (mm/ $\frac{1}{2}$ h) | b (mm ² / $\frac{1}{2}$ h) | S_c (mm) | q_o (mm/ $\frac{1}{2}$ h) | m (mm) | λ | OFV† (m/ $\frac{1}{2}$ h) |
|------|---------------|----------------------------------|--------------------------------------------|---------------|--------------------------------|-------------|-----------|------------------------------|
| 1A | 2.5 | 15.0 | ‡ | ‡ | 0.167 | 1.26 | 7.63 | 450.0 |
| 2A | 5.0 | 25.0 | ‡ | ‡ | 0.409 | 2.3 | 7.62 | 480.0 |
| 3A | 5.0 | 25.0 | ‡ | ‡ | 0.377 | 2.3 | 7.06 | 480.0 |
| 4C | 5.0 | 25.0 | ‡ | ‡ | 2.504 | 2.9 | 7.24 | 480.0 |
| 5A | 5.0 | 25.0 | ‡ | ‡ | 0.333 | 3.0 | 8.55 | 480.0 |

* Lowest estimates from sprinkler infiltration measurements.

† Measured surface flow velocities 5–18 m/ $\frac{1}{2}$ h.

‡ Inoperative in model simulations.

12. THE SIMULATION OF THE WHOLE CRIMPLE BECK BASIN

For the simulation of the whole of the Crimple Beck basin, the area was subdivided into slope and channel network elements (Fig. 3). Each slope element was simulated individually and the predicted slope outflows were routed through the channel network using the routing procedures described in section 5 and the parameters of the relationship of Fig. 10.

The analysis of topography was carried out for all the slope segments but it was not possible to run a field measurement programme to derive the parameters of all 23 segments directly. Early measurements, however, suggested that this was not necessary and parameters were assigned to some segments on the basis of similarity in land use, topography and available flow information. Representative values of measured and estimated parameters are given in Table 1 and the results of the simulation are shown in Fig. 16. The results are thought to justify further application and development of the model.

13. DISCUSSION

Since all models are simplifications of our perceptions of the real world, they cannot hope to reproduce the behaviour of the prototype in all its detail. Consequently there can be no absolute validation of any model and the term can only be applied in a relative sense with reference to some specified criterion of comparison between the real world system and the model. In fact, practical considerations may result in a model being accepted as sufficiently accurate for a given purpose without being accepted as a validated representation of the prototype.

Underlying the present project has been a concern with the prediction of the hydrological response of ungauged basins, incorporating information derived directly from the basin itself without resort to regional statistical generalization for descriptive variable of model parameters. The simulations presented here suggest that the model developed in this project provides a reasonable representation of basin behaviour using only measured and estimated parameter values. However, it was found that the

simulations were consistently better during winter conditions and the apparent seasonality in the sensitivity of the model parameters would suggest that the evapotranspiration and flow delay components of the model must yet be improved.

It is suggested that the best approach to this joint problem will be from the results of sprinkler infiltrometer experiments. It is true that such experiments are time consuming and have inherent disadvantages in terms of boundary conditions, high rainfall rates and the restrictions of single point measurement. However, provided that the assumptions (a) of soil properties which change exponentially with depth and (b) of unsaturated flow being predominantly in the vertical plane, are approximately valid, the theoretical basis for extrapolating from plot to basin scale is then capable of further development. It has also been apparent that the apparatus used could also be much improved, in particular to allow lower rates of simulated rainfall, more realistic drop sizes, and larger water reservoirs.

A number of further tests on the present model (or any improved version) are still needed. Firstly, it seems reasonable to assume on the basis of results to date, that it is not necessary to simulate all the slope segments within a given basin individually. Provided the topography and flow patterns are analysed in suitable detail, areas of similar soil conditions might be lumped together for the prediction of slope discharge. The channel network routing sub-model enables slope segment discharges to be distributed through the network in accordance with their position in the basin, for routing to the outlet. This form of lumping would greatly reduce the computing requirements of the model, but requires further validation.

Secondly, the model must be tested on other (and larger) basins, especially areas exhibiting a slower hydrological response than Crimple Beck. The size of Crimple Beck and the time step used are such that the channel network routing component cannot be considered as fully tested. It is expected, however, that the suggested procedure will remain satisfactory for larger basins, provided that the assumption of spatially uniform channel velocity remains approximately true. At larger scales, it may be necessary to allow for downstream changes in velocity as well as the simple at-a-station relationship of Equation 11.

The model presented in this paper is essentially a three way compromise between the advantages of model simplicity, the complex spatial variability of basin hydrological response and the economic limitations on field parameter measurement. The assumptions and concepts on which the model is based are seen as a satisfactory approximation to the hydrological reality under at least some circumstances, and to be capable of further development, particularly with respect to spatial variations in evapotranspiration. The model is also thought to retain enough simplicity to allow the formulation of sediment and pollutant models which recognize that overland flow is not produced uniformly over a basin.

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