Chapter 6

TANK MODEL

6.1 WHAT IS THE TANK MODEL?

M. Sugawara

The tank model is a very simple model, composed of four tanks laid vertically in series as shown in Fig. 6.1. The tanks are of uniform size, and each tank is considered to have an evaporation rate equal to the rainfall rate in the tank. The top tank is considered to have an evaporation rate equal to the rainfall rate in the top tank. If there is no water in the top tank, the rainfall is considered to be evaporated. If there is no water in the top tank, the rainfall is considered to be evaporated. If there is no water in the top tank, the rainfall is considered to be evaporated. If there is no water in the top tank, the rainfall is considered to be evaporated.
The tank model described above is applied to analyse daily discharge from daily precipitation and evaporation inputs. The concept of initial loss is not needed at each tank (except for the lowest tank) because the notion of initial loss is included in the non-linear structure caused by setting the side outlets somewhat above the bottom of each tank (except for the lowest tank).

The tank model can represent such many types of hydrographs because of its non-linear structure.

6.1.2. The Simple Linear Tank

If we move the side outlet(s) of each tank to the bottom of the tanks, we transform the model to one of the linear forms shown in Fig. 6.5a and Fig. 6.5b. Let us first consider a single tank model shown in Fig. 6.5a and Fig. 6.5b. Then, if $X(t)$ is the storage in the tank, the following equations hold:

\[
\begin{align*}
(i) \cdot x &= (i) \cdot \left( \frac{ip}{P} \right), \\
(i) \cdot x &= (i) \cdot \left( \frac{ip}{P} \right) \\
(i) \cdot x &= (i) \cdot X + (i) \cdot \frac{ip}{P}
\end{align*}
\]

Therefore,

\[
(i) \cdot x = (i) \cdot \left( \frac{ip}{P} \right) \\
\]

The following equations hold:

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\[
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\]
The constant $T$ of the tank is given by

$$T = \frac{1}{K}.$$

Regarding the time constant, we have an approximate formula that the
constant character of the basin and will be proportional to the square root of the catchment area. If so, in reality, the output decreases as the storage decreases. The constant $T$ also becomes shorter as the catchment area increases. The storage will become a function of $t$. The second and the third tanks should have a ratio of 1:5:10 or so. The second and the third tanks should have a ratio of 1:5:10 or so. The second and the third tanks should have a ratio of 1:5:10 or so. The second and the third tanks should have a ratio of 1:5:10 or so.

We can consider the time constant for each tank of the four-tank model.

Consider now the non-linear tank shown in Fig. 6.8. When the water level is lower than the middle side outlet, discharge is controlled by the linear operator with a time constant $T = 1/K$. When the water level is between the top and middle side outlets, discharge is controlled by the linear operator with time constant $1/K + K_1$), and when the water level is above the top outlet, the time constant is $1/K + K_1 + K_2$.

This shows that the simple linear tank of Fig. 6.6 is the Heaviside operator $k/(D+k)$, the first order lag system.

If the input at $t = 0$ to an empty linear tank is the $\delta$ function, i.e. $x_0 = \delta(t)$, then the output will be the exponential function $y(t) = k \exp(-kt)$, as shown in Fig. 6.7. If we draw a tangent to this exponential curve at $t = 0$, then it will cut the horizontal axis (i.e. $y$-axis) at $T = 1/K$, which is the time constant of the linear operator $k/(D+k)$.

The time constant $T = 1/K$ is the initial value at $t = 0$, then the storage in the tank will disappear after $T = 1/K$. In reality, the output decreases as the storage decreases and after time $T = 1/K$, the storage will become $1/K$. The storage will disappear after $T = 1/K$. The second and the third tanks should have a ratio of 1:5:10 or so. The second and the third tanks should have a ratio of 1:5:10 or so. The second and the third tanks should have a ratio of 1:5:10 or so. The second and the third tanks should have a ratio of 1:5:10 or so.
where \( A(km^2) \) is the catchment area. We think that for flood analysis the time unit (T.U.) should be about one third of the time constant \( T \), i.e.

\[
T.u. = 0.1S \cdot \sqrt{A/2}.
\]

Using this formula, a basin of 10 km\(^2\) would need precipitation and discharge data at a 10 minute interval, and a basin of 100 km\(^2\) would need half-hourly data. This means that small experimental basins present difficult problems.

The formula \( T = 0.1S \cdot \sqrt{A/2} \) was obtained mostly from examples of Japanese basins where the catchment areas are small and the ground surface slope is rather high. Therefore, the coefficient \( C \) might probably be larger than 0.15 for basins in large continents, perhaps 0.2 - 0.3, or so. In the case of the Chang Jiang (the Yangtze River) at Yichang, the catchment area (Sichuan basin) is about 500,000 sq. kms (about 10,000 km\(^2\)), and the time constant of the flood is about 10 days (240 hours). If we apply \( T = C \cdot \sqrt{A/2} \) to the Yangtze River at Yichang we get

\[
C = \frac{240}{(0.15 \cdot 500,000)^{1/2}} = 0.34.
\]

Even though the difference of the coefficient values between 0.15 and 0.34 is not small, when we consider the very large range of area from 10 to half a million km\(^2\), it seems reasonable to say that the time constant of flood runoff is approximately proportional to the square root of the catchment area.

6.1.3. The Evidence for the Existence of Separated Storage of Groundwater

Despite giving good results for the calculation of river discharge from rainfall in many areas, the tank model has often been considered as only a black box model. However, there is important evidence that can give physical meaning to the tank model. In Japan, many stations measure crustal tilt for earthquake forecasting. The observations are often disturbed by noise, among which the largest is the effect of rain, as shown in Fig. 6.9 and Fig. 6.10. These data were observed in a tunnel at Nakaizu (position 138°49′48.4″E, 34°1′46.4″N; altitude 263m; lithology, tuffaceous sandstone). The observations are often disturbed by noise, among which the largest is

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The original data are shown in the figures.

![Fig. 6.9. Crustal tilt NS and EW-components at Nakaizu](image)
Using precipitation input we began to use the tank model to simulate the NS-component of the tilt curve. After a while, however, we realized that, this would not work very well because the variation in the output of the tank model was much larger than that of the tilt curve; the discharge peaks were too high when compared with the peaks of the tilt curve. However, it seemed that the tilt curves could be approximated by the storage volumes of the tank model. We can approximate the NS-component by the sum of the storage in the first and the second tank, and the EW-component by the storage of the second tank; where NS means North-South and EW means East-West.

The explanation as to why the crustal tilt curve can be approximated by the storage of the tank model is that crustal tilt is affected by the weight of groundwater storage. In other words, the crust is some sort of spring balance which 'weighs' and so measures the groundwater storage (see Fig. 6.11). The first attempt to simulate the crustal tilt curve by the output of the tank model can now be seen to be unreasonable because it assumed that the discharge or mass of water in the river channel affected the crustal tilt. However, the mass of water in the river channel is negligible in comparison with the groundwater storage.

6.1.4. The Storage Type Model

It is reasonable to consider that runoff and infiltration from the ground surface layer are functions of the storage volume. Moreover, we can imagine that the relationship between runoff and storage must be of an accelerative type and that the relationship between infiltration and storage must be of a saturation type, as shown in Fig. 6.12a. Such relationships can be approximated by a tank such as Fig. 6.12b. The two curves in Fig. 6.12a can be approximated by a tank such as Fig. 6.13b. Since these relationships can be approximated by a tank, the assumption that the relationship between runoff and storage must be of an accelerative type and that the relationship between infiltration and storage must be of a saturation type is reasonable. Therefore, we can use the tank model to simulate the ground water storage.

The main goal of seismologists is to eliminate the noise caused by the ground water storage. However, the mass of water in the river channel is negligible in comparison with the discharge or mass of water in the river channel. The crustal tilt curve caused by the weight of the groundwater storage must be some sort of spring balance which 'weighs' two kinds of groundwater storage. There must be some structure that 'weighs' two kinds of storage; storage in the first and the second tank for the NS tilt and storage in only the second tank for the EW tilt. Although the form of this structure is not clearly known, we can imagine such a structure without any difficulty.

The most important point must be the separation of these two types of structure. For a long time the tank model was only an imaginary model in our minds, but now we can show that it corresponds to a real underground structure.
successful and to improve the performance we had to divide the upper storage type into two parts, one for surface runoff and another for intermediate runoff. Later, the lower tank was also divided into two tanks, one for sub-base runoff with a rather short time constant and one for stable base runoff with a long time constant (Fig. 6.15b).

In some cases, the tank with three side outlets is used as the top tank, and the tank with two side outlets is applied for the second tank. These are some sorts of variety of the usual tank modeL.

If we set many similar small-diameter evenly-spaced outlets on a tank, as shown in Fig. 6.16a, the relationship between runoff and storage is given by a parabola in the limiting case. In some cases, a tank of this type may be an effective form of the tank shown in Fig. 6.17a. However, the tank shown in Fig. 6.17a has been used as a top tank. In this case, when the storage is between $H_1$ and $H_2$, the relationship between runoff and storage is given by a parabola, and when the storage is larger than $H_2$, the relationship is given by the tangent of the parabola as shown in Fig. 6.17b.

The soil moisture structure attached to the tank model (see Fig. 6.19) is as follows:

1. Soil moisture storage has two components, the primary soil moisture storage $X_p$ and the secondary soil moisture storage $X_s$. The soil moisture storage is given by $X = X_p + X_s$. When $X_p$ is not greater than a certain value $S_1$, $X_s$ is not stored in the soil tank. When $X_p$ is greater than $S_1$, $X_s$ is stored in the soil tank.

2. The primary soil moisture storage and free water in the top tank together make a storage $X$ in the top tank. Rainfall is added to $X$ and evaporation is subtracted from $X$. When $X$ is not greater than $S_1$, the primary soil moisture storage is saturated and free water $X_f$ is stored in the top tank. When $X$ is greater than $S_1$, the primary soil moisture storage is saturated and free water $X_f$ is stored in the top tank.

The first part is called the primary soil moisture and the later part is called the secondary soil moisture. They are shown symbolically in Fig. 6.18b. As this form of tank is somewhat inconvenient to use, we set the secondary soil moisture on the side of the tank as Fig. 6.18c shows.

The soil moisture structure attached to the tank model (see Fig. 6.19) is as follows:

1. Soil moisture storage has two components, the primary soil moisture storage $X_p$ and the secondary soil moisture storage $X_s$, where the maximum capacity of each storage is $S_1$ and $S_2$, respectively. When $X_p$ is not greater than $S_1$, $X_s$ is not stored in the soil tank. When $X_p$ is greater than $S_1$, $X_s$ is stored in the soil tank. Rainfall is added to $X$ and evaporation is subtracted from $X$. When $X$ is not greater than $S_1$, the primary soil moisture storage is saturated and free water $X_f$ is stored in the top tank. When $X$ is greater than $S_1$, the primary soil moisture storage is saturated and free water $X_f$ is stored in the top tank.

The ground surface layer is considered to have a soil moisture component. In humid regions without a dry season, such as Japan, the soil moisture is nearly always saturated and so the tank model of Fig. 6.1 can give good results without the need for soil moisture. However, if we wish to consider the effect of soil moisture we must add a soil moisture component. This is shown in Fig. 6.14a. If rain occurs on dry soil, the moisture will first fill the space which is easy to occupy, i.e., the space between the moisture and the soil surface. If rain occurs on wet soil, it will fill the space between the moisture and the soil surface. If rain occurs on dry soil, the moisture will first fill the space which is easy to occupy, i.e., the space between the moisture and the soil surface. If rain occurs on wet soil, it will fill the space between the moisture and the soil surface.
Then, we can write the following equations:

\[ \frac{d}{dt} \left( \frac{S}{X} \right) = \frac{1}{X} \frac{S}{X} \]

Therefore,

\[ \frac{1}{X} \frac{S}{X} = 0 \]

When there is no rainfall but constant evaporation, \( E \), there is no free water in the top tank but there is free water in the lower tanks. When the primary soil moisture is not saturated and there is free water in the lower tanks, we can write:

\[ \frac{d}{dt} \left( \frac{S}{X} \right) = \frac{1}{X} \frac{S}{X} \]

Therefore, the soil moisture storage can be rewritten in homogeneous form as:

\[ \frac{d}{dt} \left( \frac{S}{X} \right) = \frac{1}{X} \frac{S}{X} \]

When there is no rainfall but constant evaporation, \( E \), there is no free water in the top tank but there is free water in the lower tanks. When the primary soil moisture is not saturated and there is free water in the lower tanks, we can write:

\[ \frac{d}{dt} \left( \frac{S}{X} \right) = \frac{1}{X} \frac{S}{X} \]
\[
\frac{\frac{\partial z}{\partial x}}{\frac{\partial z}{\partial y}} = \frac{\frac{\partial z}{\partial x}}{\frac{\partial z}{\partial y}} + \frac{1}{2} = \frac{1}{2} - 1
\]

Then, the second approximation is

\[
0 = \frac{\frac{\partial z}{\partial y}}{\frac{\partial z}{\partial x}} + \frac{1}{2} = \frac{1}{2} - 1
\]

and \( K_{z} \) is small, we can obtain the first approximation:

\[
\left( \frac{\frac{\partial z}{\partial y}}{\frac{\partial z}{\partial x}} \right)^{2} = \left( \frac{\frac{\partial z}{\partial y}}{\frac{\partial z}{\partial x}} + \frac{1}{2} \right)^{2} = \frac{1}{2} - 1
\]

In our experience, \( S_2 \) is much larger than \( S_1 \), and \( K_2 \) is much larger than \( K_1 \). From Fig. 6.21, we can see that the characteristic equation has two positive roots, \( k_1 \) and \( k_2 \). The large one satisfies the condition

\[
\frac{\frac{\partial z}{\partial y}}{\frac{\partial z}{\partial x}} + \frac{1}{2} = \frac{1}{2} - 1
\]

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\[
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\]

To solve the linear homogeneous equations, we put

\[
X_p = A \exp(-kt), \quad X_s = B \exp(-kt)
\]

and obtain

\[
y = \left( k - ki + k_2 \right) \left( k - k_2 \right) = k_2^2
\]

where \( X_p \) and \( X_s \) are primary and secondary soil moisture storage and \( ki \) is the positive root of the characteristic equation. If \( ki \) is much larger than \( k_2 \), then the second approximation is valid:

\[
k_2^2 = ki^2 + k_2^2
\]

Figure 6.20 shows the behavior of soil moisture storage for different values of \( E \). When \( E < k_1 \), the new origins are low, as shown in Fig. 6.20a, and when \( E > k_1 \) the new origins are high, as shown in Fig. 6.20b. Seasonal change in the level of these origins will play an important part in the behavior of soil moisture storage.
When \( i = 50, j = 250; K_i = 2, K_2 = 20 \), the values of \( k_i, k_2 \), and their first and second approximations are given as follows:

\[
\begin{align*}
k_i' &= 0.52, \\
k_i'' &= 0.51385, \\
k_2' &= 0.00615, \\
k_2'' &= 0.006228.
\end{align*}
\]

The meaning of this general solution is simple and clear. If \( CL = 0 \), we are left with the second component with a long time constant \( T_2 = \frac{1}{k_2} \) and both soil moisture storages are represented by:

\[
s_i = s_{i2} = c_{i2} \exp(-k_2 t)\]

The vector \((A, B)\), corresponding to the large characteristic value \( k_i \), can be determined by neglecting \( k \) in the equation.

\[
-kA + K_1 A + K_2 B = 0
\]

Then, we can obtain an approximate solution:

\[
A + B = 0.
\]

If we set \( A = C_1 \) and \( B = -C_1 \), we can obtain the following approximate solution:

\[
x_p = C_1 \exp(-k_i t), \quad x_s = -C_1 \exp(-k_i t).
\]

The vector \((A', B')\), corresponding to the small characteristic value \( k_2 \), which is very small, can be determined by neglecting \( k \) in the equation.

\[
-kB = K_2 A - K_2 B
\]

Then, we can obtain an approximate solution:

\[
A - B = 0.
\]

By putting \( A = C_1 \) and \( B = -C_1 \), we can obtain the following approximate solution:

\[
x_p = C_1 \exp(-k_2 t), \quad x_s = C_1 \exp(-k_2 t).
\]

The meaning of this general solution is simple and clear. If \( CL = 0 \), we are left with the second component with a long time constant \( T_2 = \frac{1}{k_2} \) and both soil moisture storages are represented by:

\[
s_i = s_{i2} = c_{i2} \exp(-k_2 t), \quad s_s = s_{s2} = c_{s2} \exp(-k_2 t).
\]

Now, we have to notice that the points of origin for \( x_p \) and \( x_s \) are \( c_i \) and \( c_s \), respectively. In Fig. 6.20. On the Pacific Ocean side of Japan, winter is a rather dry season because the relative humidity is high. According to the weather is dry, the soil moisture is high. The relative humidity decreases exponentially from the relative humidity at point 1 to point 2. In the equation:

\[
\frac{d^2 x}{dt^2} + k_1 \frac{dx}{dt} + k_2 x = 0
\]

The relative humidity at point 1 is \( x_1 \) and the relative humidity at point 2 is \( x_2 \). In the equation:

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\]

The relative humidity at point 1 is \( x_1 \) and the relative humidity at point 2 is \( x_2 \). In the equation:

\[
\frac{d^2 x}{dt^2} + k_1 \frac{dx}{dt} + k_2 x = 0
\]
6.16 The Compound Tank Model for Arid Regions

Fig. 6.22

Consider the tank model for the first zone. The input to and output from the groundwater layer (lower zone) is shown in Fig. 6.23. Following these considerations, we divide the basin into zones.

In this model we assume that, when surface runoff occurs in a zone, the lower zones are already wet and the surface runoff from the top tank of each lower zone will go directly to the lower channel. For the second, third, and fourth zones, however, the runoff will go to the corresponding tank of the next higher tank. Therefore, even if the whole basin is similar, the potential evaporation from the fourth tank does not occur from the dry areas. The actual evaporation decreases as the percentage of dry areas increases. On the contrary, during the dry season the surface runoff increases. The wet season increases with time, and the percentage of wet areas also increases with time. During the wet season, the percentage of wet areas increases with time and the mountain areas become dry. However, during the dry season, the mountain areas remain wet because they receive groundwater from the lower areas. The wet season becomes dry because the groundwater moves downward by gravity. On the other hand, a snow-covered area with an annual potential evaporation greater than the potential evaporation is covered with snow. In regions where the annual potential evaporation is greater than the annual precipitation, the snow moisture will disappear in 46 days from the saturation date. In conclusion, the soil moisture will disappear completely in a relatively short period if the potential evaporation is high. In arid regions where the annual potential evaporation is larger than the annual precipitation, mountainous areas become dry in the dry season, because groundwater moves downward by gravity. On the other hand, low areas along the river remain wet because they receive groundwater from the higher areas. When the wet season begins and there is sufficient rainfall occurs, the groundwater moves downward by gravity. Therefore, even if the whole basin is similar, the potential evaporation from the fourth tank does not occur from the dry areas. The actual evaporation decreases as the percentage of dry areas increases. On the contrary, during the dry season the surface runoff increases. The wet season increases with time, and the percentage of wet areas also increases with time. During the wet season, the percentage of wet areas increases with time and the mountain areas become dry. However, during the dry season, the mountain areas remain wet because they receive groundwater from the lower areas. The wet season becomes dry because the groundwater moves downward by gravity. On the other hand, a snow-covered area with an annual potential evaporation greater than the potential evaporation is covered with snow. In regions where the annual potential evaporation is greater than the annual precipitation, the snow moisture will disappear in 46 days from the saturation date.
the tank model or when compared to $Q(j)$, the observed

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through the tank model itself can give some sort of time lag (the simple

Time Lag

result by trial and error.

rank of Fig. 6.25. Using several tanks of such a type, we can attain a good
enhanced capacity and this condition can be simulated with the storage type
water storage or inundation occurs above a restriction because of the limited

Fig. 6.26

Some other sort of storage type tank such as shown in Fig. 6.26.

Fig. 6.25

Chamber Reservoir

Deformation of the Hydrograph by Water Storage or Flooding Caused by

Effect of Irrigation Water Intake

In most Japanese river basins, there are wide paddy fields which use large
volumes of irrigation water. In many cases, the irrigation water intake is

Effect of Irrigation Water Intake, Deformation of

Hydrograph by Gorge, etc.

some sort of storage type tank such as shown in Fig. 6.26.

Effect of Irrigation Water Intake

In most Japanese river basins, there are wide paddy fields which use large
volumes of irrigation water. In many cases, the irrigation water intake is

Water storage or inundation occurs above a restriction because of the limited

Fig. 6.25

Fig. 6.26

Percentages should be:

47.7% : 4.2% : 1% = 1

42.3% : 3% : 1% = 1

29.5% : 2.5% : 1% = 1

25.7% : 2% : 1% = 1

18.5% : 3% : 1% = 1

17.8% : 1% : 1% = 1

15.5% : 1% : 1% = 1

12.3% : 1% : 1% = 1

10.1% : 1% : 1% = 1

8.9% : 1% : 1% = 1
It is better to keep these coefficients as simple numbers, for example if \( A_0 = 0.001 \), then it is easier to make it 0.2 or 0.3.

\[
\begin{align*}
D_1 &= 0.001, \\
C_0 &= 0.001, \\
B_1 &= 0.001, \\
A_2 &= 0.001,
\end{align*}
\]

The Initial Model

The initial model shown in Fig. 6.27a is the result of the above considerations. After the peaks, if the discharge rate is lower than the descending rate of discharge, the model coefficients of \( A_0, B_1, C_0, D_1 \) are reduced. If the discharge rate is higher than the descending rate of discharge, the model coefficients of \( A_0, B_1, C_0, D_1 \) are increased. For the first trial, an initial tank model may be assumed; an example of such an initial model may be assumed:

\[
\begin{align*}
A_0 &= 1 / 3, \\
B_1 &= 2 / 3, \\
C_0 &= 1 / 3, \\
D_1 &= 0.001.
\end{align*}
\]

The tank model is non-linear and the mathematical treatment is nearly useless for non-linear problems. Therefore, the author could not use mathematics for the tank model calibration and the only solution was to use the trial and error method of numerical calculations. In 1951 when the author first applied the simple tank model for runoff analysis there were only a few computers in Japan and the author was not able to use one. Without mathematical solutions and with no computer, the numerical calculations necessitated long and hard labour. However, the human mind is always curious and the labourious numerical calculations became not so boring but rather interesting as experience and judgement were built up in the author's brain. Gradually, calibration of the tank model became not so difficult and relatively easy. Usually the experience and judgement were useful in finding the initial model coefficients. However, the human mind is always curious and the labourious numerical calculations become not so difficult and relatively easy. Usually the experience and judgement were useful in finding the initial model coefficients. However, the human mind is always curious and the labourious numerical calculations become not so difficult and relatively easy.

6.2. HOW TO CALIBRATE THE TANK MODEL?

6.2.1. Trial and Error Using Subjective Judgement

It is better to keep these coefficients as simple numbers, for example if \( A_0 = 0.266 \), then it is better to make it 0.25 or 0.3.

The tank model is non-linear and mathematics is nearly useless for non-linear problems. Therefore, the author could not use mathematics for the tank model calibration and the only solution was to use the trial and error method of numerical calculations. In 1951 when the author first applied the simple tank model for runoff analysis there were only a few computers in Japan and the author was not able to use one. Without mathematical solutions and with no computer, the numerical calculations necessitated long and hard labour. However, the human mind is always curious and the labourious numerical calculations became not so boring but rather interesting as experience and judgement were built up in the author's brain. Gradually, calibration of the tank model became not so difficult and relatively easy. Usually the experience and judgement were useful in finding the initial model coefficients. However, the human mind is always curious and the labourious numerical calculations become not so difficult and relatively easy. Usually the experience and judgement were useful in finding the initial model coefficients. However, the human mind is always curious and the labourious numerical calculations become not so difficult and relatively easy.
Find the Worst Point of the Running Model and Adjust the Parameter Values.

How to Determine the Positions of Side Outlets.

In Japan, experience shows that if it rains less than 15mm after about 15 dry days then there will be no change in river discharge. The position of the lower side outlet of the top tank, $H_{A1} = 15$, is determined from this experience.

Also we know that when it rains more than 50mm during the rainfall, the discharge will increase greatly. The position of the upper side outlet of the top tank, $H_{A2} = 40$, is determined in this way considering also the water loss from the top tank by infiltration and runoff during the rainfall.

In the tank model of Fig. 6.27, the side outlets of the second and third tank are set at $H_B = H_C = 15$. These are determined to be similar to $H_{A1} = 15$, but without such good reasoning, because $H_B$ and $H_C$ are not as effective as $H_{A1}$. The effect of $H_B$ or $H_C$ appears when the runoff component from the second or the third tank vanishes under dry condition.

Examples are shown in Fig. 6.28 of hydrographs from tank models with $H_C = 15$, $H_C = 30$ and $H_C = 100$. These hydrographs were obtained under the following conditions.

1. The entire hydrograph output of the running model must be plotted five times as $Y_5, Y_4 + Y_5, Y_3 + Y_4 + Y_5, Y_2 + Y_3 + Y_4 + Y_5$, and $Y_1 + Y_2 + Y_3 + Y_4 + Y_5$ (see Fig. 6.27b). Comparing the five calculated hydrographs with the observed hydrograph, we can judge which component is the worst. In early trials, there may be many bad components. But the shape of the hydrograph may also be a problem. For the hydrographs described above, we can change the volume of each runoff component.

2. How to Adjust the Shape of Hydrograph.

If the worst point is that the runoff from the top tank is too small, there may be two ways to correct for this. One way would be to make $A_1$ and $A_2$ larger, and another way would be to make $A_0$ smaller. However, the best way is to both make $A_1$ and $A_2$ larger and to make $A_0$ smaller, i.e. multiply $A_1$ and $A_2$ by $k_1 (k_1 > 1)$ and divide $A_0$ by $k_2$. If $0 < k_1 < k_2 < 1$, for example $k_1 = 1.5, k_2 = 1.25, k_3 = 1\frac{1}{4}$, the output from the top tank will become smaller. The output from the second tank or the third tank can be adjusted in the same way.

In the case where judgment shows that the base discharge is too small, the method described above cannot work because the fourth tank has no bottom outlet. Therefore, we must increase the water supply to the fourth tank by making $C_0$ larger. However, this would decrease the runoff from the third tank. Hence, the adjustment to make the base runoff larger is made by increasing $C_0, C_1$, and $A_0$ as follows.

- $C_0 = k_1 C_0$, $C_1 = k_2 C_1$, $A_0 = k_3 A_0$
- $k_1 > 1, k_2 > 0$ and $k_3 > 0$
- $k_1 = 1 + k, k_2 = 1 + k/2, k_3 = 1 + k/4$,
- or $k_1 = k, k_2 = k/2, k_3 = k/4$.

In Japan, experience shows that the lower side outlet of the top tank, $H_{A1} = 15$, is determined from this experience. Experience shows that it rains less than 15mm after about 15 dry days.

How to Determine the Positions of Side Outlets.

For the tank model of Fig. 6.28, the positions of the side outlets are determined as follows.

- The initial storage of the third tank is given as $S_3 = 100$mm, $S_3 = 200$mm, and $S_3 = 300$mm, as shown in Fig. 6.27a.
- The position of the side outlet of the top tank, $H_{A1} = 15$, is determined from this experience.
- Also we know that when it rains more than 50mm during the rainfall, the discharge will increase greatly. The position of the upper side outlet of the top tank, $H_{A2} = 40$, is determined in this way considering also the water loss from the top tank by infiltration and runoff during the rainfall.

In the case where judgment shows that the base discharge is too small, the method described above cannot work because the fourth tank has no bottom outlet. Therefore, we must increase the water supply to the fourth tank by making $C_0$ larger. However, this would decrease the runoff from the third tank. Therefore, we must increase the water supply to the third tank from the second tank. In such a case the side outlet of the third tank is determined as $H_C = 15, 30, 100$, as shown in Fig. 6.28. The position of the side outlet of the third tank is determined as $H_C = 15, 30, 100$, as shown in Fig. 6.28.
Some Sort of Balance and Harmony is Important for the Tank Model

Why the author wished to develop the automatic calibration program?

6.2.2 Automatic Calibration Program (Hydrograph Comparison Method)

According to the results, the result will improve after a few trials.

While considering this point, it may be better to start again from a different initial model. The shape of the Q-t curve may resemble or exhibit a problem of obtaining a profile on the basement. By drawing the slope of the observed hydrograph after a peak flow, we can get an idea of the shape of the initial model. We think that such balance and harmony are important for the initial model.

We have the following relationships:

\[ \frac{A}{V_0} = B_0 = C_1 = C_2 = C_3 = \frac{k}{3} \]

The coefficients of the initial tank models shown in Fig. 6.27 and Fig. 6.30 are

In some basins, where the balance and harmony do not exist, it may be better to start again from a different initial model. If the new initial model is more suitable for such regions and instead, the tank models of Fig. 6.30 are recommended for such regions. In these cases, the base discharge is very high and the surface is very small. The initial tank model of Fig. 6.27 is not suited for such regions and instead, the tank models of Fig. 6.30 are recommended as the initial model.

In these cases, the fourth unit may be replaced by the type shown in Fig. 6.29.

Volcanic Areas.

In regions where the land surface is covered by thick volcanic deposits, the infiltration ability of the land surface is very high, the surface runoff is small and the base discharge is very large. The initial tank model of Fig. 6.27 is suitable for such regions and the tank models of Fig. 6.30 are recommended as the initial model.

Fig. 6.29.

1975, the author retired as director of the institute and had more free time for research. However, the following year, the author was warned by the medical authorities that he might soon die from disease. This opinion turned out to be wrong but the scare convinced the author that it was his duty to develop an automatic method of calibrating the tank model so that knowledge of calibration techniques should not be lost with his death.

Once the work had begun, it turned out to be rather easy. It was the type of problem which stimulated the author's way of thinking and after a few weeks, he had succeeded in developing an automatic method of calibrating the tank model so that the result obtained through this method was very similar to the result obtained through the manual method. However, the following year, the author was warned by the medical authorities that he might soon die from disease. This opinion turned out to be wrong but the scare convinced the author that it was his duty to develop an automatic method of calibrating the tank model so that knowledge of calibration techniques should not be lost with his death.

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The program is a trial and error method carried out by computer. A rough outline of the program is as follows:

1. Start with an initial model.
2. Divide the whole period into five subperiods, each subperiod corresponding to one of the five runoff components.
3. Compare the calculated hydrograph with the observed hydrograph for each subperiod, and define the criteria \( RQ(i) \) and \( RD(i) \) (\( i = 1, 2, \ldots, 5 \)) where \( RQ(i) \) is the criterion for volume and \( RD(i) \) is the criterion for the hydrograph shape of the \( i \)-th subperiod.
4. Adjust the coefficients of the tank model according to the criteria \( RQ(i) \) and \( RD(i) \).
5. The hydrograph derived by the adjusted model is compared with the observed, and an evaluation criterion \( CR \) is calculated.
6. The next trial is made using the adjusted model.
7. The automatic calibration procedure will usually finish after several iterations of a few trials and the model of the least criterion \( CR \) is considered to be the best model.

### Division of the Hydrograph into Five Subperiods

Division of the hydrograph into five subperiods so that in the \( i \)-th subperiod the \( i \)-th runoff component is the most important.

### Introduction of Criteria \( RQ(i) \) and \( RD(i) \)

These rules were determined after some modification and improvements to simplify the programming. These rules were determined after some modification and improvements to simplify the programming.

Figure 6.31. These rules were determined after some modification and improvements to simplify the programming.

These rules were determined after some modification and improvements to simplify the programming.

### Evaluation of the Effectiveness of the \( i \)-th Runoff Component

Evaluation of the effectiveness of the \( i \)-th runoff component must be made in the period in which the \( i \)-th runoff component is most important. To accomplish this, the entire hydrograph is divided into five subperiods as follows:

- **Subperiod 1:** Days on which \( Y_1 \) is most important belong to subperiod 1.
- **Subperiod 2:** When \( Y_1 + Y_2 > C \), \( C \) is a constant (usually set to 0.1) belongs to subperiod 2.
- **Subperiod 3:** When \( Y_1 + Y_2 + Y_3 > C \).
- **Subperiod 4:** When \( Y_1 + Y_2 + Y_3 + Y_4 > C \).
- **Subperiod 5:** Otherwise.

Division of the hydrograph into five subperiods so that in the \( i \)-th subperiod the \( i \)-th runoff component is the most important.

### The Automatic Calibration Procedure

The automatic calibration procedure will usually finish after several iterations of a few trials and the model of the least criterion \( CR \) is considered to be the best model.

The program is a trial and error method carried out by computer.

### Tank Model

- \( Y_1 \) \( \rightarrow \) \( Y_2 \)
- \( Y_3 \) \( \rightarrow \) \( Y_4 \)
- \( Y_5 \) \( \rightarrow \) \( Y_6 \)
- \( Y_7 \) \( \rightarrow \) \( Y_8 \)
- \( Y_9 \) \( \rightarrow \) \( Y_{10} \)
The iterative feedback described above, the tank model would converge to a

We would normally expect that, starting from an initial model and using

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

By solving these equations we can derive the following feedback formulae:

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

Similarly, if \(A_0\) is adjusted to

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

should be adjusted to

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

coefficients should be adjusted to

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

coefficients where \(A_0\), \(A_1\), and \(A_2\) are adjusted coefficients.

For the second tank, it is better to adjust the coefficients \(A_0\) and \(A_1\) by

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
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D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

should be adjusted to

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

coefficients where \(A_0\), \(A_1\), and \(A_2\) are adjusted coefficients.

For the third tank, we cannot adjust the discharge amount by modifying

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

coefficients where \(A_0\), \(A_1\), and \(A_2\) are adjusted coefficients.

For the fourth tank, we cannot adjust the discharge amount by modifying

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

coefficients where \(A_0\), \(A_1\), and \(A_2\) are adjusted coefficients.

Later on, an improvement to \(R(1)\) and \(R(2)\) was made based on the

Feedback formula:

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

Thus, if \(R(1) = 1\) or \(R(2) = 1\) then we use the following feedback formula:

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

By solving these equations we can derive the following feedback formulae:

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

Similarly, if \(A_0\) is adjusted to

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

should be adjusted to

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

coefficients where \(A_0\), \(A_1\), and \(A_2\) are adjusted coefficients.

For the second tank, it is better to adjust the coefficients \(A_0\) and \(A_1\) by

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
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Similarly, if \(A_0\) is adjusted to

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A &= \frac{A_0}{R(0)} \\
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D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

should be adjusted to

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

coefficients where \(A_0\), \(A_1\), and \(A_2\) are adjusted coefficients.

For the third tank, we cannot adjust the discharge amount by modifying

\[
\begin{align*}
A &= \frac{A_0}{R(0)} \\
B &= \frac{B_0}{R(0)} \\
C &= \frac{C_0}{R(0)} \\
D &= \frac{D_0}{R(0)} \\
E &= \frac{E_0}{R(0)}
\end{align*}
\]

coefficients where \(A_0\), \(A_1\), and \(A_2\) are adjusted coefficients.
Effect of RD(I) must be halved

When the author was calibrating the tank model by his trial and error method of subjective judgement, the best model was also determined subjectively. However, for the trial and error procedures by computer, an evaluation criterion was developed.

We select two criteria for feedback: the other criteria are eliminated by the tank model. RD(I) and RO(I) which are most distant from 1 are only considered by which we select two criteria within RD(I) and RO(I) of which we select two.

In calibrating the tank model by the trial and error method there is one important principle: we must always concentrate our attention on only one weak point of the model. If a model has many weak points, we cannot improve it properly. Therefore, we concentrate our attention on only one weak point, which we think is the best method.

Values of RD(I) and RO(I) which are close to 1 should not be used for feedback.

In the early stages of developing the automatic calibration program, feedback procedures using RD(I) and RO(I) did not give good results. This was probably caused by feedback of RD(I) and RO(I) for values near to 1. Such criteria which are close to 1 include little information but are mostly composed of noise, and although the feedback effects of each of these criteria are small, the total effect is very unreliable. Therefore, the feedback of RD(I) and RO(I) for values near to 1 should be neglected.

As a rule of thumb, the time lag between the peak discharge of observed and calculated discharge is never greater than 1. This is about one day. In the case of Fig. 6.33a, RO(i) and RO(I) (especially RO(I)), are greatly affected when J = 1, 2, 3, 4, 5 (especially RO(I)). The peak discharge in period 5 is very small, RO(I) is mainly composed of noise, and, accordingly, it is unreliable. Therefore, the feedback of RD(I) should be neglected.

As the time constant of the fourth tank is very long, the slope of calculated discharge in period 5 is very small. RO(I) is mainly composed of noise, and, accordingly, it is unreliable. Therefore, the feedback of RD(I) should be neglected.

In some cases, the values of RO(I) and RO(I) (especially RO(I)) may show values very different from 0. To avoid such extreme values we replace them equal to 1.

Values of RD(I) and RO(I) which are close to 1 should not be used for feedback.

Finally, the RD(I) and RO(I) of some criteria which are not very reliable may still be used for feedback by putting them equal to 1.
One of the best known points of view is that the hydrological cycle of different tanks is very shy different initial models will get different final models and it is very difficult to judge from simulation outputs which tanks are the best models. However, this was misplaced optimism. Starting from different initial models, the model with the fewest parameters (i.e., the one with the smallest number of parameters) is expected to be the most realistic model. Therefore, deciding which model to use is necessary to develop the most realistic calibration program.

## Evaluation Criterion

It is necessary to determine an evaluation criterion to decide on the best tanks.

**Mean Square Error (MSE)**

\[ \text{MSE} = \frac{1}{J} \sum_{j=1}^{J} \left( \frac{Q_{E}(j) - Q_{D}(j)}{Q_{D}(j)} \right)^2 \]

where:
- \( Q_{E}(j) \) is the observed discharge at day \( j \)
- \( Q_{D}(j) \) is the predicted discharge at day \( j \)
- \( J \) is the number of days

The criterion obtained is

\[ \text{MSELQ} = \frac{1}{J} \sum_{j=1}^{J} \left( \frac{\ln Q_{E}(j) - \ln Q_{D}(j)}{\ln Q_{D}(j)} \right)^2 \]

Finally, the evaluation criterion \( CR \) is defined by

\[ CR = \text{MSE} + \text{MSELQ} \]

For the overall evaluation of the model, it may be better to use the modified MSELQ as follows:

\[ CR = \frac{1}{J} \sum_{j=1}^{J} \left( \frac{\ln Q_{E}(j) - \ln Q_{D}(j)}{\ln Q_{D}(j)} \right)^2 \]

In the case shown in Fig. 6.33a, \( Q_{E} \) shows large errors for days \( j \) and, in the case of Fig. 6.33c, \( Q_{E} \) shows large errors for days \( j \) and \( j+1 \). Since such cases occur frequently, both \( \text{MSEQ} \) and \( \text{MSELQ} \) are too high. Even if the difference between \( Q_{E}(j) \) and \( Q_{D}(j) \) is large in such cases, we can say that both programs are similar. Any model which has a large error in \( j \) and a small error in \( j+1 \) is not useful for the purpose of this study.

## Number of Parameters

In the case of the error criterion:

\[ \frac{\partial^2 \text{MSELQ}}{\partial Q_{D}(j) \partial Q_{D}(j+1)} = 0 \]

Therefore, in the case of the error criterion:

\[ \frac{\partial^2 \text{MSELQ}}{\partial Q_{D}(j+1) \partial Q_{D}(j+1)} = 0 \]

If the difference between \( Q_{E}(j) \) and \( Q_{D}(j) \) is small compared to the relative error, then the modified MSELQ may be defined as follows:

\[ \text{MSELQ} = \frac{1}{J} \sum_{j=1}^{J} \left( \frac{\ln Q_{E}(j) - \ln Q_{D}(j)}{\ln Q_{D}(j)} \right)^2 \]

### Uniqueness of the Solution

In some cases of linear regression, the solution is unique. However, if the correlation coefficient is high, the solution may be unique. In other cases, the solution may not be unique. Therefore, the number of iterations should be kept below a certain number. In the computer program, the tank model coefficients are adjusted by the RQ(I)'s and RD(I)'s. The two which are most distant from 1 within these eight RQ(I)'s and RD(I)'s are selected, and these two are used to adjust the tank model coefficients. Therefore, after four iterations it is probable that all the effective RQ(I)'s and RD(I)'s will have been used for adjustment, the adjusted model will become good, and all the RQ(I)'s and RD(I)'s will have become 1. Therefore, the number of iterations should be kept below a certain number. In the computer program, the tank model coefficients are adjusted by the RQ(I)'s and RD(I)'s. Therefore, as the number of iterations increases, the tank model will have become good, and it is possible to use the computer program to develop calibration programs. However, there must be some reason why the number of iterations should be kept below a certain number.

### Error Propagation

The number of iterations of the model should be kept below a certain number. In the computer program, the tank model coefficients are adjusted by the RQ(I)'s and RD(I)'s. Therefore, as the number of iterations increases, the tank model will have become good, and it is possible to use the computer program to develop calibration programs. However, there must be some reason why the number of iterations should be kept below a certain number.

### Evaluation Criterion

The evaluation criterion is necessary to decide on the best tank model. While the evaluation criterion is necessary to decide on the best tank model, it is probably better to use the natural logarithm of discharge, as follows:

\[ CR = \text{MSEQ} + \text{MSELQ} \]

This means that \( CR \) is the mean square of the relative error and is a measure of the goodness of the model.
The tank model is some sort of approximation of the runoff phenomena. There must therefore be many ways of approximating the result, i.e. there cannot be only one unique solution.

6.2.3. Automatic Calibration Program by Duration Curve

Comparison Method

Why comparison of duration curve became necessary?

In 1975 and 1976, the author visited the Upper Nile Region around the Lake Victoria, and had the chance to analyse the runoff of several basins. This region is located just under the equator and the rainfall is always local showers covering small areas. Therefore, even if there are several rainfall stations in the basin, this will not be enough to catch the rainfall over the basin. In some cases, nearly all stations may record heavy rain but the discharge from the basin is small. On the other hand, the rainfall from the basins in the basin will not be enough to cause the rainfall over the basin to exceed the average rainfall. Therefore, even if the rainfall is above average, the discharge from the basin is small. This region is located under the equator and the rainfall is always local showers covering small areas. Lake Victoria and the other lakes in the region are located near the equator, and the rainfall is above average.

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On the duration curve of calculated discharge derived from the working

The duration curve of calculated discharge derived from the working

After the division of the duration curve into subsections the definition of

The duration curve of calculated discharge derived from the working

Why comparison of duration curve became necessary?

In 1975 and 1976, the author visited the Upper Nile Region around the

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This region is located just under the equator and the rainfall is always local showers covering small areas. Lake Victoria and the other lakes in the region are located near the equator, and the rainfall is above average.

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For the duration curve comparison method we have to introduce new evaluation criteria as follows:

\[
MSEDC = \frac{1}{N} \sum_{i=1}^{N} \left( Q_i - Q_{\text{calc}} \right)^2
\]

where \(Q_i\) and \(Q_{\text{calc}}\) are the observed and calculated discharges, respectively.

The method is resistant to divergence because the duration curve becomes more reliable than each single criterion individually.

The final evaluation criterion used in the duration curve comparison method is given by

\[
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where \(CR\) is the sum of the mean square error of the calculated and observed discharges.

For the duration curve comparison method we have to introduce new evaluation criteria as follows:

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RQ(I) = \frac{1}{L} \sum_{i=1}^{L} \left( Q_i - Q_{\text{calc}} \right)
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where \(Q_i\) and \(Q_{\text{calc}}\) are the observed and calculated discharges, respectively.

Some characteristics of the duration curve comparison method:

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- The final evaluation criterion used in the duration curve comparison method is given by

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where \(CR\) is the sum of the mean square error of the calculated and observed discharges.

Feedback formulae

The feedback formulae are just the same as those used in the hydrograph comparison method. However, we can not understand the exact reason why the effect of \(RQ(I)\) must be halved.

In the hydrograph comparison method, we halved the effect of \(RQ(I)\) to reduce the effect of \(RQ(I)\) because of its reliability caused by the large noise effect. In the present case, \(RQ(I)\) is applied to the feedback formulae without dividing it by 2, because we do not expect the feedback formulae to bring good results. However, we can not understand the exact reason why the effect of \(RQ(I)\) must be halved.

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where \(CR\) is the sum of the mean square error of the calculated and observed discharges.
The goodness of fit of duration curves is probably more meaningful than the goodness of fit of hydrographs. If we apply both methods to some basin and compare the resulting hydrographs, the hydrograph comparison method usually shows a slightly better result than the duration curve comparison method. This should be obvious because, in the hydrograph comparison method, the feedback procedures are made to give a good fit of the observed hydrographs, while, in the duration curve comparison method, the feedback procedures are made to give a good fit of the duration curves and usually, the evaluation criteria CRDC is smaller than CRHY, i.e. the duration curves fit better than the hydrographs.

We suppose that a good fit of duration curves is probably more meaningful than a good fit of hydrographs both from hydrological and practical points of view. It is clear, that for practical uses (say for hydroelectric power generation), a good fit of the duration curves is more meaningful than a good fit of the hydrographs. The hydrological point of view is not so obvious but can be developed as follows: There are many peaks in a hydrograph. In any particular storm rainfall data observed at some stations will show smaller values than the unknown real mean areal rainfall, and in some other storm the contrary case will occur. Therefore, in one case the observed peak will be greater than the calculated one and, in the other case, the observed peak will be smaller than the calculated one. Accordingly, when a pattern between the observed and the calculated one is considered, the peaks in the calculated curve will be higher than the unknown real mean areal rainfall in some stations and lower than the unknown real mean areal rainfall in other stations. Therefore, the goodness of fit of the hydrographs is more or less determined by the calculation of the hydrographs power generation equation (say for hydroelectric power generation). It is clear, that for practical purposes, a good fit of the duration curves is probably more meaningful than a good fit of hydrographs.

We support that a good fit of duration curves is probably more meaningful than a good fit of hydrographs.
The soil moisture structure has two time constants $T_1$ and $T_2$. The short time constant $T_1$ shows how the difference between the relative wetness of primary and secondary soil moisture decreases with time as they approach the equilibrium state. The long time constant $T_2$ shows how the total soil moisture decreases with time under drying conditions. Therefore, $T_1$ and $T_2$ have entirely different hydrological effects, can be considered as independent factors and can be calibrated one by one.

Usually, $S_2$ is much greater than $S_1$, and $K_2$ is much greater than $K_1$, and so we can derive rough but simple approximate relations as follows:

$$T_1 = \frac{S_1}{K_2},$$
$$T_2 = \frac{S_2}{K_1}.$$

For simplicity, we put $T_1 = \frac{S_1}{K_2}$ and $T_2 = \frac{S_2}{K_1}$ so that $T_1$ and $T_2$ are approximately equal to the short time constant $T_1$ and the long time constant $T_2$, respectively.

We can then calibrate $T_1$ and $T_2$ in the following four steps:

**First step**

To determine the value of $T_1$, we start from some initially assumed values of $S_1$ and $K_2$. For example, $S_1$ is changed to $0.95 S_1$, $S_1$ and $1.05 S_1$, and $K_2$ is changed to $0.95 K_2$, $K_2$ and $1.05 K_2$. This will change the value of $T_1$ about 10%. Trials are made, similar to the case of HB, and the value of $T_1$ is determined so as to give the minimum value of $CR$. As $T_1$ is approximately equal to the short time constant $T_1$, which has important hydrological meaning, it is important to determine the value of $T_1$ accurately.

**Second step**

Keeping the value of $T_1$ constant, the values of $S_1$ and $K_2$ are changed simultaneously by 10%. In this way, $S_1$ and $K_2$ are changed.

**Third step**

The value of $T_1$ is calibrated in the same way as in the first step.

**Fourth step**

The values of $S_2$ and $K_1$ are calibrated in the same way as in the second step.

The snow model and some other important factors

Division into zones

When winter comes, snow begins to deposit on high elevation areas and then spreads to lower areas. When spring comes, the snow deposit begins to melt first in low elevation areas and then moves up the elevation range of the basin. Therefore, in order to calculate snow deposit and melt, the number of zones need not be too large; usually, it is sufficient to divide the basin into a few zones with equal elevation intervals. When winter comes, snow begins to deposit on high elevation areas and when spring comes, snow begins to deposit on lower elevation areas.

Temperature decrease with elevation

Equation (1) is derived from the assumption that the mean temperature decreases with elevation per 1,000m is about 5°C-6°C. We can derive the rate of temperature decrease with elevation per 1,000m in most cases, the precipitation and snow deposit are governed by air temperature, and so the rate of temperature decrease with elevation is one of the most important factors in the snow model. Usually, the temperature decrease per 1,000m in the lowest zone is located in the lowest zone and there are no stations in the intermediate zones. In our approach, we approximate the decrease in the mean temperature with elevation to be 5.5°C per 1,000m in the lowest zone. In most cases, the temperature decrease with elevation is about 6°C per 1,000m, so in these cases, the precipitation and snow deposit are governed by air temperature, and so the rate of temperature decrease with elevation is introduce.

Temperature decrease with elevation

Equation (2) is derived from the assumption that the mean temperature decreases with elevation per 1,000m is about 5°C-6°C. We can derive the rate of temperature decrease with elevation per 1,000m in most cases, the precipitation and snow deposit are governed by air temperature, and so the rate of temperature decrease with elevation is one of the most important factors in the snow model. Usually, the temperature decrease per 1,000m in the lowest zone is located in the lowest zone and there are no stations in the intermediate zones. In our approach, we approximate the decrease in the mean temperature with elevation to be 5.5°C per 1,000m in the lowest zone. In most cases, the temperature decrease with elevation is about 6°C per 1,000m, so in these cases, the precipitation and snow deposit are governed by air temperature, and so the rate of temperature decrease with elevation is introduce.

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temperature over most of the basin, after neglecting the effects of elevation. This must be due to the following two reasons: One, the station is usually set at a convenient place for observation, i.e. in or near a village or some place which is convenient to stay or visit. Such a place is usually warmer than the rest of the basin. The other reason might be that places where people live become warmer because of the effect of human activities. In some cases, it seems to be better to assume that the precipitation in the highest zone is the same as that in the next highest zone. For example, in the case of four zones the precipitation's are given as follows:

\[ (l + PO) - P, \]
\[ (l + PO + CP(M) - PD) - P, \]
\[ (l + PO + 2CP(M) - PD) - P, \]
\[ (l + PO + 2CP(M) - PD) - P. \]

Precipitation increase with elevation. However, as the area of the fourth zone is not large, the effect of this assumption is not so large. Moreover, if it rains, we can assume that the temperature of the rain water is equal to the air temperature. Then, the snowmelt by rainwater is \( PT/80 \), where \( P \) is the daily precipitation.

Therefore, the maximum amount of daily snowmelt is given by

\[ SM = (l + PO + P)(l - 1) - CP(M) - PD - P. \]

The snowmelt constant is governed by air temperature and we may assume that the amount of snowmelt in a day is proportional to the mean air temperature. Moreover, if it rains, we can assume that the temperature of the rain water is equal to the air temperature.

The assumption is not so large. However, as the area of the fourth zone is not large, the effect of this assumption is not so large. Moreover, if it rains, we can assume that the temperature of the rain water is equal to the air temperature. Then, the snowmelt by rainwater is \( PT/80 \), where \( P \) is the daily precipitation.

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The weights of precipitation stations should be determined in such cases due to micro-meteorological conditions. The rainfall almost useless for discharge forecasting, in spite of the large differences in some cases. A station situated near the center of a basin may be exampled in some cases. A station situated near the center of a basin may be far off from their geographical position. But the geographical conditions. For principle, the weights of precipitation stations should be determined.

**Figure 6.36**

### 6.3.2 Precipitation Factor CP(M)

In snowy basins, precipitation increases with elevation shows a large seasonal change. Correspondingly, in most non-snowy basins, the seasonal change is not so important for the calculation of discharge.

In principle, the weights of precipitation stations should be determined, not by their geometrical positions, but by the meteorological conditions. For example, in some cases, a station situated near the center of a basin may be almost useless for discharge forecasting, while another one situated close to the boundary of a basin, in the basin center, may be almost useless for discharge forecasting. However, if the two stations are located near to each other, their weights should be almost equal. If the station A is located on A, the weight of A should be almost equal.

To calculate runoff from the tank model, the precipitation as rain and the snowmelt water are summed up in each zone, and the annual precipitation in each zone is multiplied by the area of each zone. The sum is then put into the tank model.

The Thiessen polygon method is logical.

Though the Thiessen polygon method has been widely used for a long time, it is illogical and, in reality, cannot give results. We can clearly see the defect of the method by considering two stations close to the boundary of a basin, as in Fig. 6.36. Using the Thiessen polygon method, the weight of A is very small and that of B very large. However, if the two stations are located closely, their weights should be almost equal. If station A is moved to A', the weight of A' becomes very large and that of B very small. If the Thiessen polygon method were logical, such an absurdity would not occur.

### 6.3.3 Weights of precipitation stations

In some special cases, CP(M) shows very large values in some specific months. It seems to be curious, but probably from some specific conditions. The equation of CP(M) shows very large values in some specific months. It is better to adjust or change the initial tank model if necessary. We can also make an empirical expression of CP(M).

We can find the empirical formula of CP(M) by comparing the calculated and observed monthly discharges in parallel. By comparing the calculated and observed monthly discharges in parallel, we can derive empirical expressions of CP(M).

### 6.3.4 Approximate value of CP(M)

We can estimate the approximate value of CP(M) from which we can estimate the precipitation factor CP(M). In the same way, we can derive the equation of AV for AV(1).

\[
(1 - d) S / (1 - d) S + (1 - d) d S = (1 - d) S
\]

### 6.3.5 Approximate volume in the 1-th zone

The approximate volume in the 1-th zone is

\[
SD / d = AV S / 365
\]

Together, the real value of AV decreases when the area of the 1-th zone is unknown. If AV is known, then SD = PD / 365. If PD is given, then SD = AV S / 365. The approximate volume in the 1-th zone is

\[
SD / d = AV S / 365
\]
Tank Model / 6

Determining the weights of precipitation stations by factor analysis

Therefore, the daily runoff derived from the precipitation at each station are very close to zero in that daily precipitation at station in near the basin are usually smaller in each other. In other words, they are highly correlated.

To derive the calculated discharge from precipitation at each station, the calculated discharge derived from the precipitation at each station should be in a small area around the station, may have a weak relationship with the areal precipitation over the whole basin.

Determining the weights of precipitation stations by factor analysis

First, we make monthly or half-monthly sums of the observed discharge and the calculated discharge derived from the precipitation at the I-th station, where I = 1, 2, ..., N. Let the derived time series of monthly or half-monthly observed discharge be y(n), and xi(n) (I = 1, 2, ..., N), respectively.

Next, these time series are normalized by dividing them by the square root of the mean of square of each component: i.e., (\frac{\sum y(n)^2}{N})^{1/2} and (\frac{\sum x_i(n)^2}{N})^{1/2}. Let the normalized time series be \hat{y}(n) and \hat{x}_i(n), respectively.

Then, the problem is to find the best approximation of \hat{y}(n) by the linear form \sum w(I) \hat{x}_i(n), where the coefficients w(I) is the weight of the precipitation station I. This problem can be solved easily by the method of least squares.

When NP is the number of precipitation stations, the problem is to determine w(I)'s which make \sum \hat{y}(n) - \sum w(I) \hat{x}_i(n)^2 minimum. To determine w(I), \sum \hat{y}(n) - \sum w(I) \hat{x}_i(n)^2 is differentiated partially with respect to w(I).

Putting -\frac{\partial}{\partial w(I)} (\sum \hat{y}(n) - \sum w(I) \hat{x}_i(n)^2) = 0, we get the following equation:

\sum \hat{x}_i(n) \frac{\partial}{\partial w(I)} \hat{x}_i(n) = \frac{\partial}{\partial w(I)} \hat{y}(n)

where \sum \hat{x}_i(n) \hat{x}_j(n) = A_{ij}, \frac{\partial \hat{y}(n)}{\partial w(I)} = B_I (I = 1, 2, ..., NP), and A_{ij} = \frac{\sum \hat{x}_i(n) \hat{x}_j(n)}{N}, B_I = \frac{\sum \hat{x}_i(n) \hat{y}(n)}{N}.

We had hoped that the solution of this equation would give good weights. However, the solution of this equation is often meaningless, showing positive and negative numbers with large absolute values. In most cases, the determinant of the matrix (A_{ij}) is very close to zero, and the roots of the equation \sum \hat{x}_i(n) \hat{x}_j(n) w(I) = B_I (I = 1, 2, ..., NP), where A_{ij} = \frac{\sum \hat{x}_i(n) \hat{x}_j(n)}{N}, B_I = \frac{\sum \hat{x}_i(n) \hat{y}(n)}{N} are usually similar to each other. In other words, they are highly correlated.

When the basin is very small and there are no woods, no steep mountain, no lake, etc., that prevent the establishment of precipitation stations, then we can measure the approximate value of mean areal precipitation over the basin by setting many precipitation stations uniformly over the basin. It is some sort of sampling survey. However, in almost all cases, such a precipitation measurement is impossible, and so, to measure the mean areal precipitation over the basin, some sort of sampling survey is necessary. However, in almost all cases, the mean areal precipitation over the basin is unknown. When we have observed discharge and mean areal precipitation over the basin is unknown, the problem of determining the weights of precipitation stations is meaningless.

The mean areal precipitation over the basin is unknown.

Even though the mean areal precipitation over the basin is unknown, the mean observed discharge is known. Therefore, there is a need to transform the observed discharge into precipitation. The problem is to derive the mean areal precipitation over the basin from the observed discharge. To our regret, this problem seems to be impossible. The transformation from precipitation to discharge is an integral-like operator. Most of the rain is stored in the ground before it is turned into discharge. For example, the tank model is some sort of a non-linear incomplete integral. Therefore, the inverse operator which will transform precipitation into discharge is in integral-like form. The transformation from precipitation into discharge is impossible. The transformation from observed discharge into precipitation is also impossible.

The problem then is to derive the mean areal precipitation over the basin from the observed discharge. To do this, we have to start the preliminary runoff analysis using some weights, usually, equal. If some precipitation station is found to be not representative in the course of runoff analysis, it is better to make the weight of this station small or neglect it. After the model has become good enough, the calibration of the weights of the weights with the derived runoff measurements of the stations will make the weights of the stations small or neglected. If the model has become good enough, the calibration of the weights of the weights with the derived runoff measurements of the stations will make the weights of the stations small or neglected. If the model has become good enough, the calibration of the weights of the weights with the derived runoff measurements of the stations will make the weights of the stations small or neglected.
6 / M. Sugawara

also similar to each other, consequently, the time series of calculated
monthly or half-monthly discharges, \( x_i(n) \) (\( i = 1, 2, \ldots, NP \)) are similar to each
other, as are the normalized time series \( X_i(n) \). Hence, when we consider the

\[ \text{time series } (X_i(n)) \text{ as a vector } X_i, \] the angle of intersection between any
two of these vectors is small. Therefore, all \( A_{ij} \) are close to 1, and

\[ \det(A) \text{ shows a very small value.} \]

Moreover, if the working model has been calibrated well enough, the
daily runoff derived from the precipitation at each station will be similar to
the observed runoff. Accordingly, every vector \( X_i \) (\( i = 1, 2, \ldots, NP \)) is similar
to \( Y \), i.e. the angle of intersection between \( X_i \) and \( Y \) is also small, and

\[ \text{therefore, all } A_{ij} \text{ are close to } 1. \]

To get a meaningful solution to such an equation the method of factor
analysis was applied. The matrix \( A \) was transformed into diagonal form
by orthogonal transformation. The derived diagonal elements are called the
characteristic values of \( A \). By neglecting those characteristic values
which are close to zero we can get a meaningful solution. Such a treatment
is mathematics, not hydrology and so the author will stop the description
here. It is not difficult to learn the method of factor analysis from some text
books on mathematical mathematics.

7.2. The Concept of Runoff Formation on Repletion of Storage

7.3. INTRODUCTION

R.J. Zhao and X. R. Liu

Chapter 7

THE XINANJIANG MODEL

7.1. INTRODUCTION

Soil, or indeed any porous medium, possesses the ability of holding
water indefinitely against gravity to a certain amount of water constituting a storage.
This is sometimes called "field moisture capacity". By definition, water
held in this storage cannot become runoff and the storage can be depleted
only by evaporation or the transpiration of the vegetation. Hence the
soil moisture content of the aeration zone of any porous medium, is the
soil moisture available for repletion. The factor that controls the
repletion of soil moisture is the net rainfall.)

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The Xinanjiang model was developed in 1973 and published in 1980 (Zhao et al., 1980). Its main feature is the concept of runoff formation on repletion of storage, which means that runoff is not produced until the soil moisture content of the aeration zone reaches field capacity, and thereafter runoff equals the rainfall excess without further loss. This hypothesis was first proposed in China in the 1960s, and much subsequent experience supports its validity for humid and semi-humid regions. According to the original formulation, runoff so generated was separated into two components using Horton's concept of a final, constant, infiltration rate. Infiltreated water was assumed to go to the groundwater storage and the remainder to surface, or storm runoff. However, evidence of variabilty in the final infiltration rate, and in the unit hydrograph assumed to connect the storm runoff to the discharge from each sub-basin, suggested the necessity of a third component. Guided by the work of Kirkby (1978) an additional component, interception, was provided in the model in 1980. The modified model is now

\[ \text{REFERENCES} \]

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